

# DGD Days 2017: Talks & Abstracts

## Wednesday, 4th October 2017

### **Franz Pedit: Conformal flows minimizing the bending energy for curves and surfaces.**

We will discuss a flow on closed planar curves and compact surfaces in Euclidean 3-space, both defined on a certain finite codimension submanifold in the Hilbert space of square integrable functions/densities. The flow moves a closed planar curve to a multiply wrapped circle or figure eight. There is evidence that genus zero surfaces in Euclidean 3-space of Willmore energy  $W < 16\pi$  flow to a round sphere ( $W=4\pi$ ) or a round sphere with a sphere bubbling off ( $W=8\pi$ ). The overall viewpoint is to replace the non-linear heat flow analysis for the gradient flow of the bending energy (which a priori experiences derivative loss) by standard ODE theory on Banach spaces, linear elliptic analysis, and the infinite dimensional geometry of the configurations spaces our flows move on.

### **Grzegorz Jablonski: Persistence in sampled dynamical systems**

The goal of our research is to embed persistence in the computational analysis of dynamical systems. We narrow our research to the case of the homomorphism induced in homology by a continuous map on a topological space. It is characterized up to conjugacy by its Jordan form, therefore the inference of the eigenvectors from sampling of the map allows for partial reconstruction of the Jordan normal form.

### **Nada Sissouno: Compressed sensing and dynamical systems**

One task in the field of dynamical systems is the determination of the analytic expression of the system or good approximative formulas. Deriving numerically such formulas often requires a large number of observations. To overcome this problem the use of compressed sensing techniques has been proposed recently. The applicability of these methods is not studied to the full extend, though. Our goal is to derive a proper analysis of the requirements. In this talk we will present challenges and first steps in achieving this goal. This is joint work with Christian Kühn and Felix Krahmer.

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## Thursday, 5th October 2017

### **Stephan Tillmann: Computing trisections of 4-manifolds**

Gay and Kirby recently generalised Heegaard splittings of 3-manifolds to trisections of 4-manifolds. A trisection describes a 4-dimensional manifold as a union of three 4-dimensional handlebodies. The complexity of the 4-manifold is captured in a collection of curves on a surface, which guide the gluing of the handlebodies.

After defining trisections and giving key examples and applications, I will describe an algorithm to compute trisections of 4-manifolds using arbitrary triangulations as input. This results in the first explicit complexity bounds for the trisection genus of a 4-manifold in terms of the number of pentachora (4-simplices) in a triangulation.

This is joint work with Mark Bell, Joel Hass and Hyam Rubinstein.

### **Arseniy Akopyan: Two circles and only a straightedge**

We answer a question of David Hilbert: given two non-intersecting circles it is not possible in general to construct their centers using only a straightedge. On the other hand, we give certain families of pairs of circles for which such construction is possible.

### **Boris Springborn: Discrete uniformization and hyperbolic polyhedra**

Two triangle meshes are considered discretely conformally equivalent if they are (a) combinatorially equivalent and (b) the lengths of corresponding edges are related by scale factors associated to the vertices. This simple notion leads to a surprisingly rich and practical theory of discrete conformal maps. It has one defect: The question of existence of solutions to mapping problems seems intractable. In this talk, I will explain how to extend the notion of discrete conformality to non-equivalent triangulations and get existence results from a variational principle. Hyperbolic geometry plays a major role: Discrete conformal mapping problems are equivalent to realization problems for ideal hyperbolic polyhedra with prescribed intrinsic metric.

### **Michael Kraus: On the structure-preserving discretization of noncanonical Hamiltonian systems**

Many systems of fluid dynamics and plasma physics have a Hamiltonian structure. Preserving this structure in the course of discretization is a nontrivial problem. Most of these systems do not possess a simple, canonical Hamiltonian structure, where the system can be described in terms of a constant symplectic form and conjugate coordinates, like most systems studied in classical mechanics. Instead, their equations are either formulated in terms of Poisson brackets or a noncanonical symplectic form, which depends on the dynamical variables, usually in a nontrivial, nonlinear way. For such systems, there are no standard methods available like the many that are known for the canonical case, most prominently symplectic and multisymplectic integrators. In this talk we will give an overview of recent efforts on the development of Hamiltonian discretization methods for the Vlasov-Maxwell system, magnetohydrodynamics and reduced kinetic models like guiding centre dynamics.

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## **Friday, 6th October 2017**

### **Johannes Wallner: Material-minimizing forms and structures**

3-dimensional structures in building construction and architecture are realized with conflicting goals in mind, and it would be very beneficial if both optimization and side conditions involving statics and geometry could play a role already in early stages of design. We discuss a step towards this goal. By combining the classical work of Maxwell, Michell, and Airy with differential-geometric considerations we obtain a geometric understanding of „optimality“ of freeform shells and discrete versions of such shells. It turns out that total absolute curvature plays an important role.

### **Albert Chern: Inside Fluids: Clebsch Maps for Visualization and Processing**

In 1859, Alfred Clebsch introduced Clebsch variables that represent a fluid state through maps. From a differential geometric perspective, these maps contain valuable information about the velocity field. For example, vortex lines are level lines of the vorticity Clebsch map. This makes Clebsch maps useful for visualization and fluid dynamics analysis. In this work we study spherical Clebsch maps, which are particularly attractive. Elucidating their geometric structure, we show that such maps can be found as minimizers of a Ginzburg-Landau energy. In the end I will also talk about the insight how spherical Clebsch map plays a role in Madelung's hydrodynamic form of quantum mechanics.

### **Anton Nikitenko: A discrete topological approach to questions in stochastic geometry**

Classical Morse theory studies topology of manifolds with functions on them. Discrete Morse theory is a not completely standardized way of applying similar observations for discrete simplicial or CW-complexes. Several widely used and well-known complexes, like Delaunay or Čech complexes, have a canonical function defined on their cells: it maps a cell to its appropriately defined radius. On the other hand, these complexes are also objects of interest in stochastic geometry, and despite very intense research, many properties of random Poisson–Delaunay and related complexes remain unknown. Building a link between discrete topology and probability, we were able to show how discrete Morse theory helps to find the distributions of radii and the expected numbers of the cells in different dimensions for several different random complexes. We will discuss new results and some open questions.

### **Sara Krause-Solberg: A tractable approach for one-bit compressed sensing on manifold**

Classical Compressed Sensing (CS) deals with reconstructing some unknown vector from few linear measurements in high dimension by additionally assuming sparsity, i.e., many entries are zero. Recent results guarantee recovery even when just signs of the measurements are available (one-bit CS). A natural generalization of classical CS replaces sparse vectors by vectors lying on manifolds having low intrinsic dimension  $d$ .

In this talk a tractable strategy to solve one-bit CS problems for data lying on manifolds is proposed. Moreover, we will see that a fairly good recovery with high probability is guaranteed if the number of measurements is of order  $d \log(d)$ , i.e., the number of measurements grows asymptotically almost linearly in  $d$  as in the classical case. This is joint work with Mark Iwen, Felix Kraemer and Johannes Maly.