DGD | SFB TRR 109 | Discretization in Geometry and Dynamics

Calendar 2023

CONTENTS & REFERENCES

JANUARY – JUNE

January

Discrete Nets via Checkerboard Patterns. In the checkerboard pattern approach a principal net is characterized by all faces being planar and every second face being a rectangle. A dual net is a parallel net where corresponding rectangles are similar but with reversed orientation. A principal net that is also dualizable is an isothermic net. This property is preserved under Möbius transformations and dualization which allows the easy construction of the Enneper surface.

Reference.

Dellinger, F. (2022). Discrete Isothermic Nets Based on Checkerboard Patterns. arXiv: 2205.01971 [math.DG]

February

Circular Nets with Spherical Parameter Lines. Circular nets in Euclidean 3-space with the combinatorics of the square grid provide a discretization of curvature-line parametrized surfaces. Surfaces with spherical curvature lines are classical surfaces that motivate the study of circular nets with spherical parameter lines. The research project compares and contrasts the properties of these circular nets to those of surfaces with spherical curvature lines.

Reference.

Bobenko, A.I., Fairley, A.Y. (in preparation). Circular nets with spherical parameter lines and terminating Laplace sequences.

March

Hyperbolic Incircular Nets. An incircular net is a two-parameter family of straight lines with the combinatorics of the square grid, such that every elementary quadrilateral admits an incircle. Incircular nets can be considered in spherical, Euclidean and hyperbolic geometry. A unified treatment can be achieved by utilizing non-Euclidean Laguerre geometry, showing the rich geometric structure of these nets and close connection to (discrete) confocal conics.

Reference.

Bobenko, A.I., Lutz, C.O.R., Pottmann, H., Techter, J. (2021). *Non-Euclidean Laguerre geometry and incircular nets*. SpringerBriefs. ISBN: 978-3-030-81846-3. x+137 p.

April

Discrete CMC Surfaces. This research project focuses on the construction of discrete surfaces with constant mean curvature built from touching discs. The surfaces are S-isothermic. They also possess touching spheres that intersect the discs orthogonally in their points of contact. Discrete CMC surfaces are recovered from their discrete Gauß map, which is given by an orthogonal ring pattern on a sphere.

Reference.

Bobenko, A.I., Hoffmann, T., Smeenk, N. (in preparation). Constant mean curvature surfaces from ring patterns: geometry from combinatorics.

May

Nonrigid Isometric Paneling. In this project we represent free forms by panel arrangements that are manufacturable by isometric bending of surfaces made from a small number of molds, in particular from constant Gauß curvature surfaces. Computations are based on a discrete model of isometric mappings between surfaces. Further topics are connections of the paneling problem with Killing vector fields, designing and actuating isometries, and curved folding in the double-curved case.

Reference.

Jiang, C., Wang, H., Ceballos Inza, V., Dellinger, F., Rist, F., Wallner, J., Pottmann, H. (2021). Using isometries for computational design and fabrication. *ACM Trans. Graph.*, 40(4), 42:1 – 42:12

June

Discrete Ellipsoids. The construction of pairs of discrete principal nets that are discrete analogues of ellipsoids is based on a novel discretization, which has recently been applied to the construction of discrete confocal coordinate systems. The diagonals of the quadrilaterals of any such discrete ellipsoid form two families of closed planar polygons, which are discrete analogs of the circular cross sections of an ellipsoid. They admit a one-parameter family of deformations such that the discrete circles are preserved.

Reference.

Huang, B., Schief, W.K., Techter, J. (in preparation). A canonical discrete analogue of the classical circular cross sections of ellipsoids and their isometric deformation.

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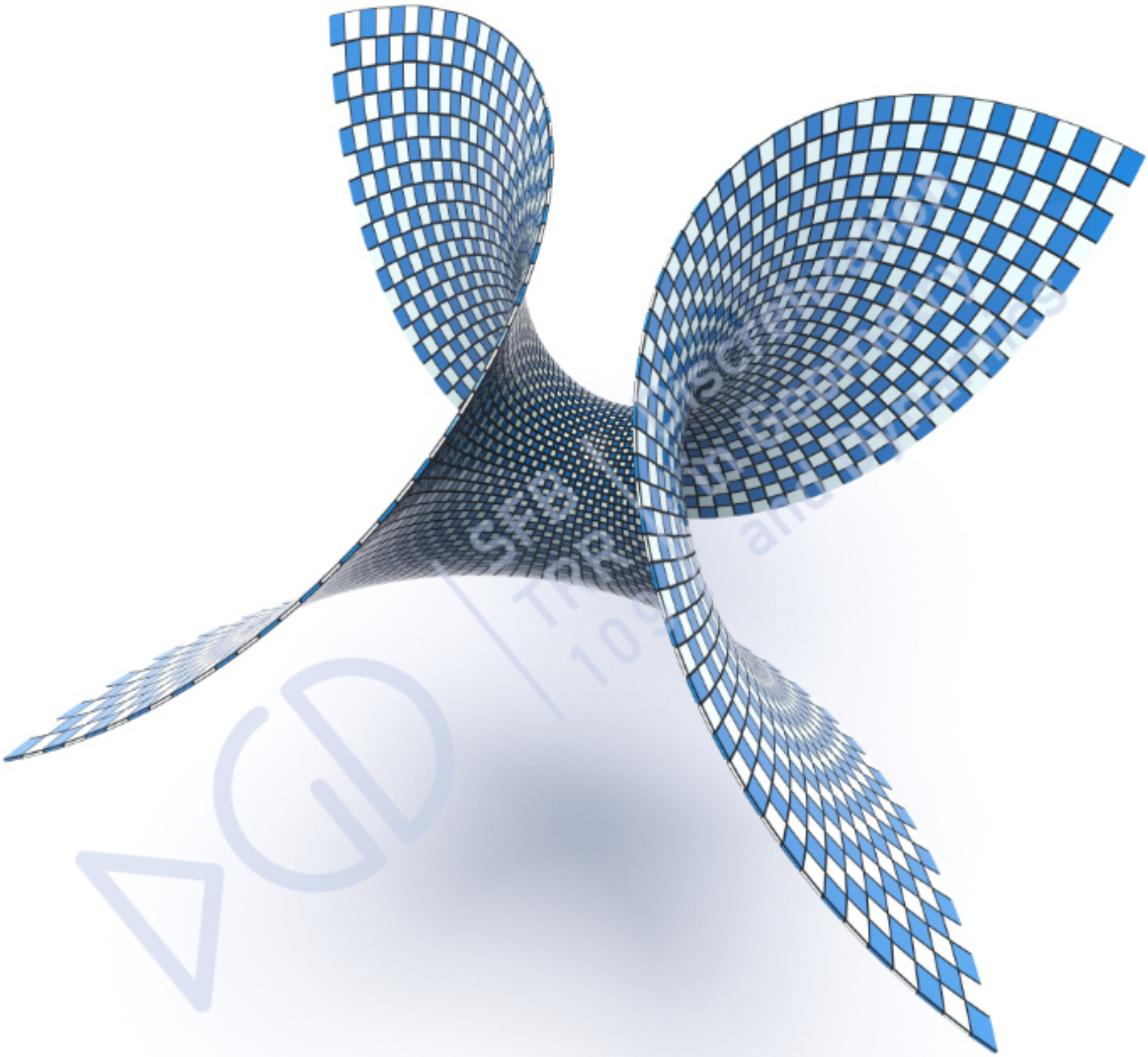

JANUARY

Discrete Nets via Checkerboard Patterns. This image shows an isothermic parametrization of the Enneper surface based on checkerboard patterns. It is constructed by applying a Möbius transformation to the unit square grid followed by dualization.

Contributor. *Felix Dellinger*

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
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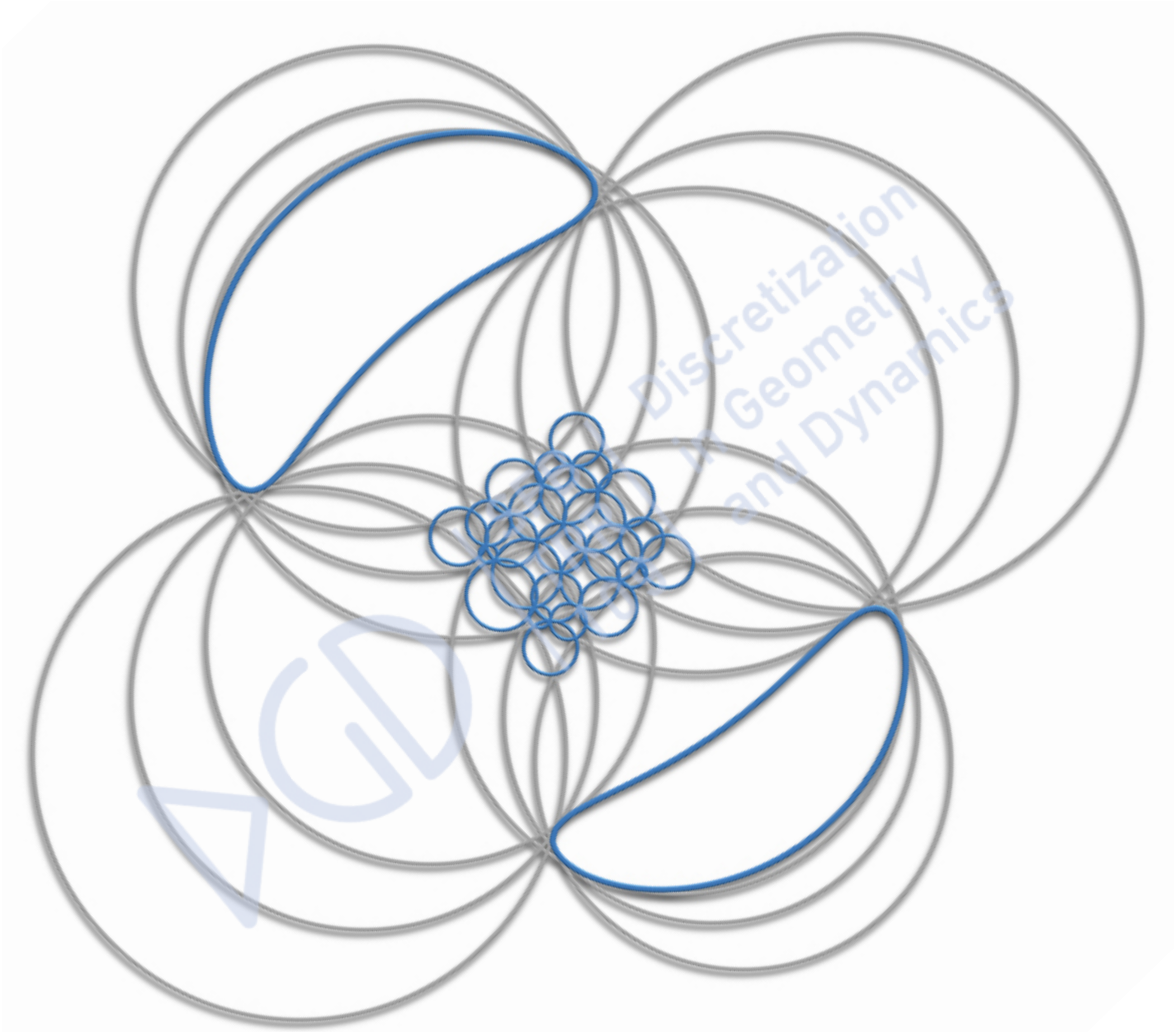

FEBRUARY

Circular Nets with Spherical Parameter Lines. A circular net (blue, middle) in the Euclidean plane with two families of circular parameter lines (gray). For each parameter line, the vertices of the net are contained in a circle. These circles envelop a bicircular quartic (blue curve with two components). Each of the circles is twice tangent to the bicircular quartic.

Contributors. *Alexander I. Bobenko, Alexander Y. Fairley*

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
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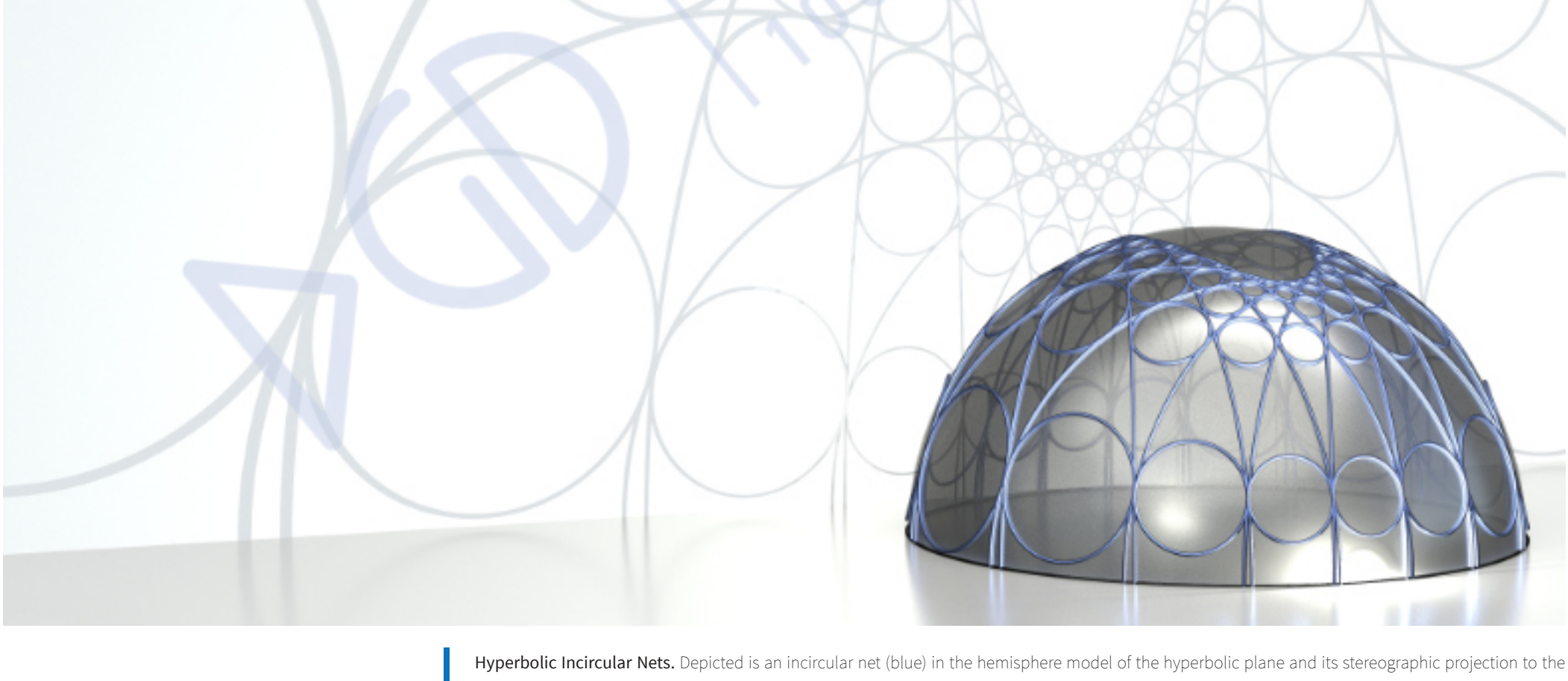

MARCH

Hyperbolic Incircular Nets. Depicted is an incircular net (blue) in the hemisphere model of the hyperbolic plane and its stereographic projection to the upper half-plane model (shadows). Such a net is given by a two-parameter family of hyperbolic lines with the combinatorics of the square grid. All lines are tangent to a hyperbolic ellipse and every elementary quadrilateral is inscribed by a hyperbolic circle.

Contributors. *Alexander I. Bobenko, Carl O. R. Lutz, Helmut Pottmann, Jan Techter*

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
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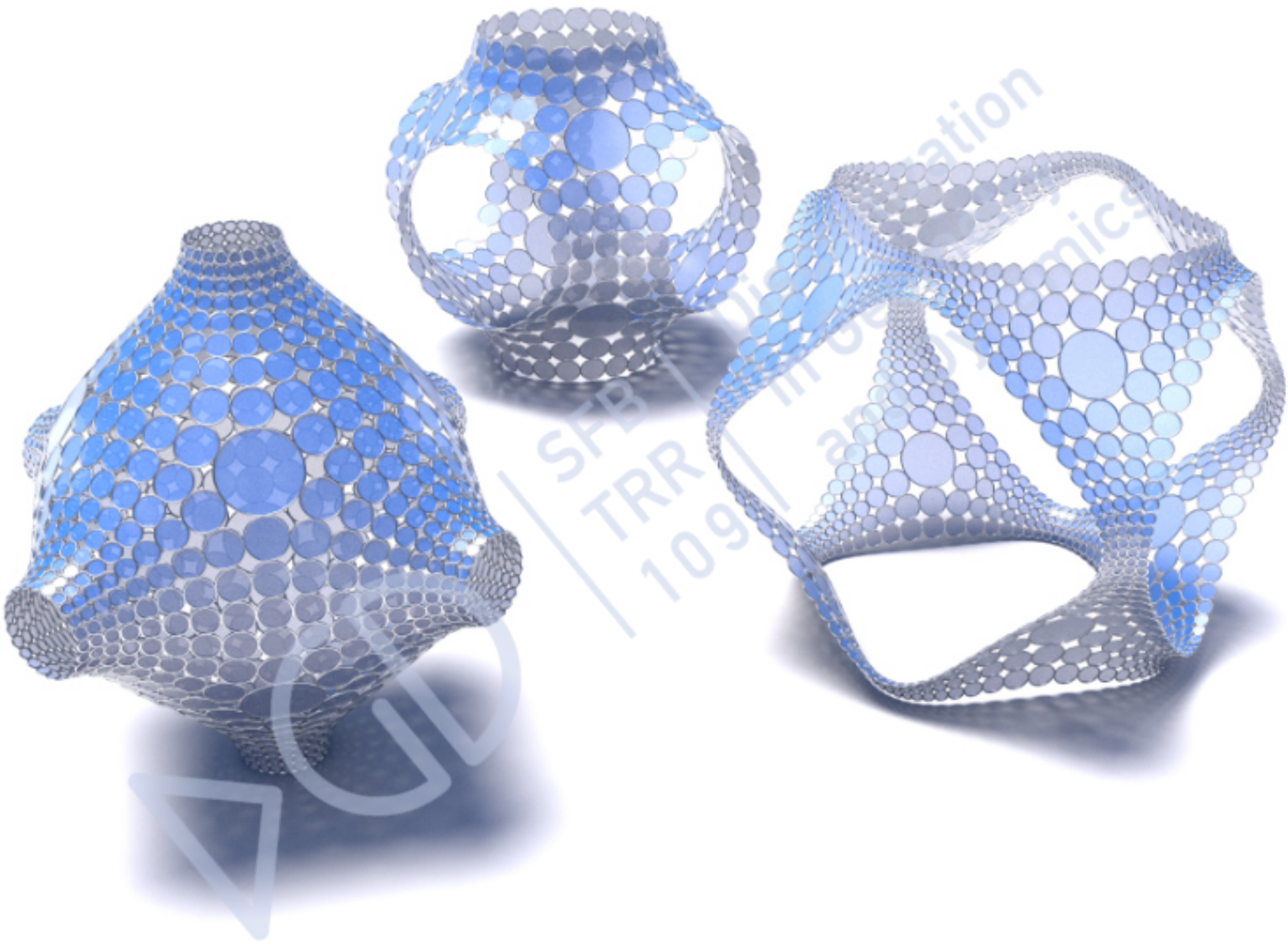

APRIL

Discrete CMC Surfaces. This image shows three surfaces from a 1-parameter family of discrete constant mean curvature (CMC) surfaces with fixed topology. The continuous parameter of the family is given by the value of the mean curvature. All surfaces are comprised of touching discs, which can be computed from the corresponding orthogonal ring patterns on the sphere.

Contributors. *Alexander I. Bobenko, Tim Hoffmann, Nina Smeenk*

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
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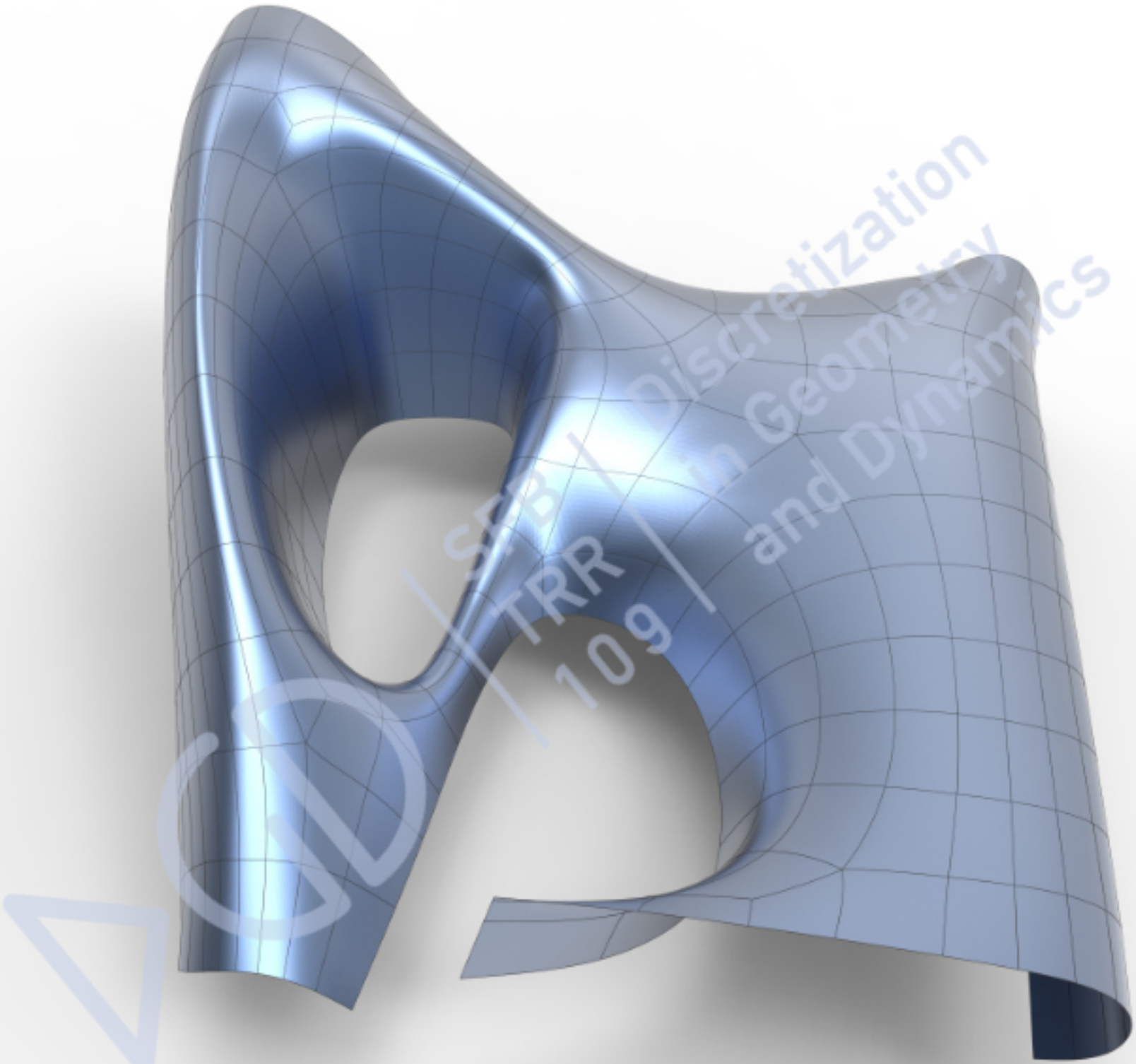

MAY

Nonrigid Isometric Paneling. This image shows a design by Zaha Hadid Architects, which has been approximated by a surface whose Gauß curvature is constant within prescribed regions (faces of the indicated mesh). The Gauß curvature is allowed to assume only a small number of discrete values. In fact, in most panels it is zero, i.e., most panels can be isometrically developed into the Euclidean plane.

Contributors. *Caigui Jiang, Hui Wang, Victor Ceballos Inza, Felix Dellinger, Florian Rist, Johannes Wallner, Helmut Pottmann*

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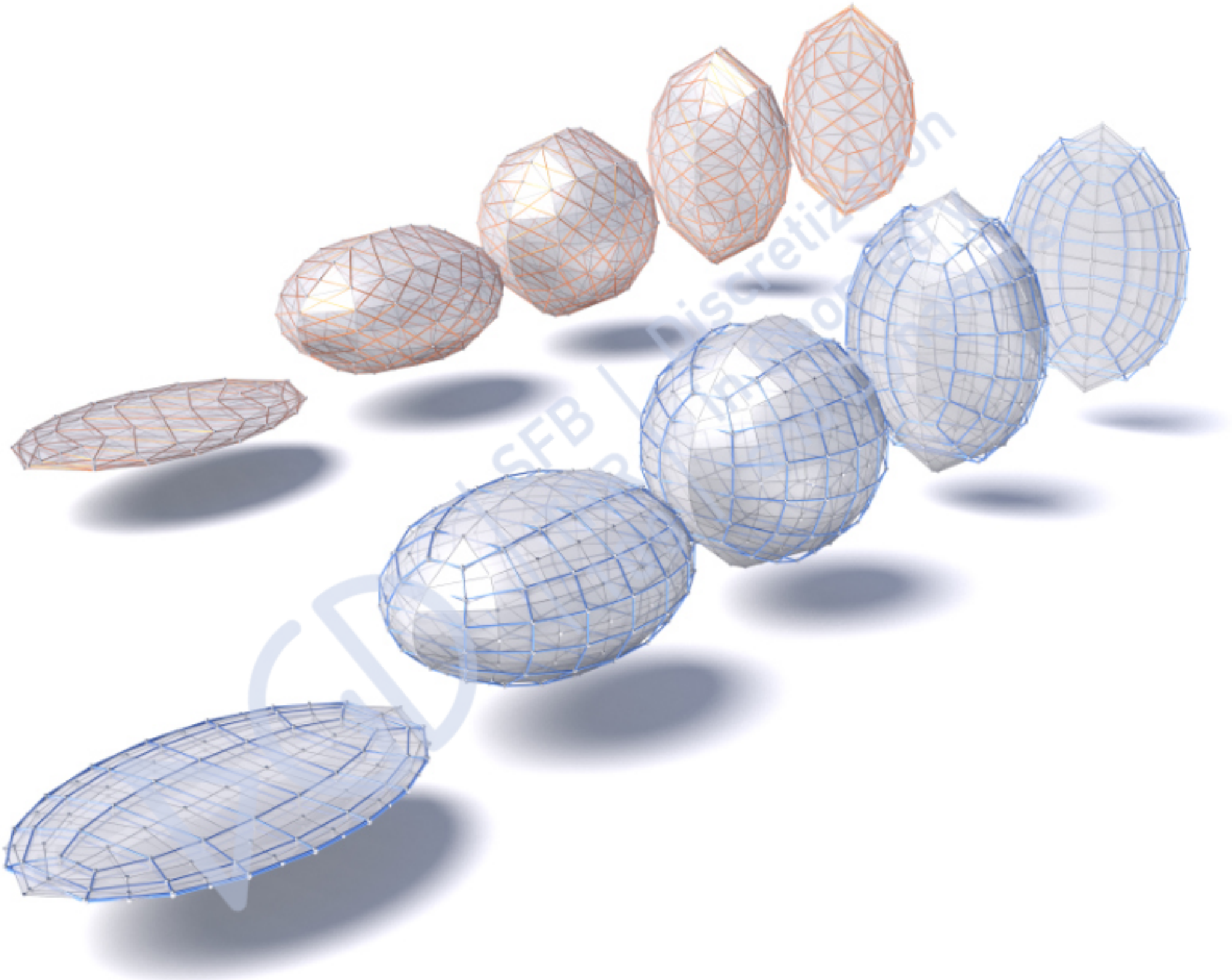

JUNE

Discrete Ellipsoids. Discrete ellipsoids arising as pairs of discrete principal nets with planar quadrilaterals and orthogonal dual edges (front). The diagonals of the planar quadrilaterals form discrete circles (rear). The shown deformation preserves the discrete circles.

Contributors. *Boris Huang, Wolfgang K. Schief, Jan Techter*

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
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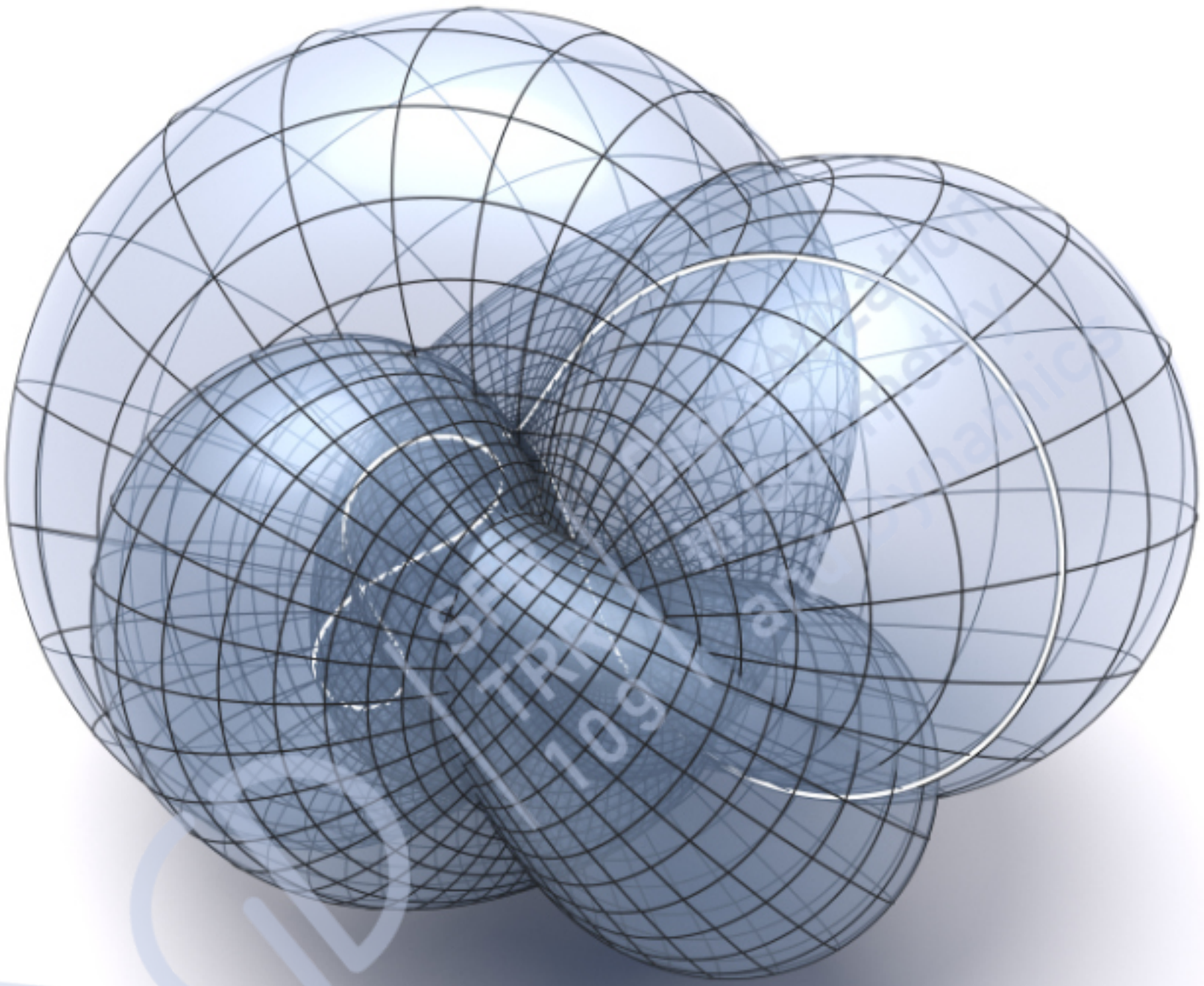

JULY

Isothermic Tori. This image shows an isothermic torus with 180°-rotational symmetry. It has one family of planar curvature lines. Two of the planar curvature lines are highlighted. This isothermic torus gives rise to an example of a Bonnet pair, i.e., two non-congruent immersed tori that are related by a mean curvature preserving isometry.

Contributors. *Alexander I. Bobenko, Tim Hoffmann, Andrew O. Sageman-Furnas*

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
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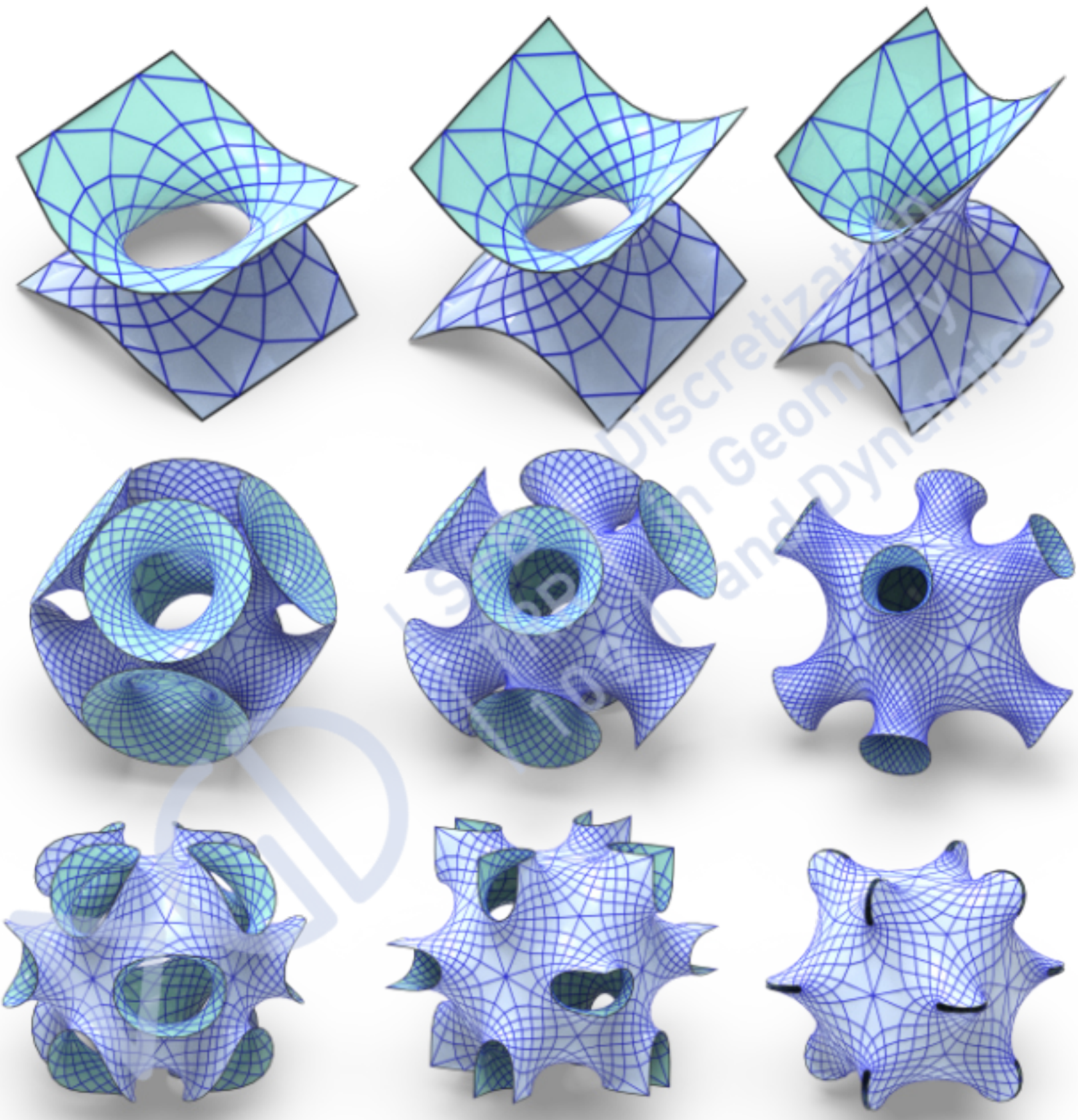

AUGUST

Discrete Surfaces with a Constant Ratio of Principal Curvatures. These surfaces are obtained by changing the right angle between asymptotic directions in discrete minimal surfaces (middle column) to another constant angle. Surprisingly, this is possible while keeping the combinatorics of the asymptotic net.

Contributors. *Hui Wang, Helmut Pottmann*

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
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7	8	9	10	11	12	13
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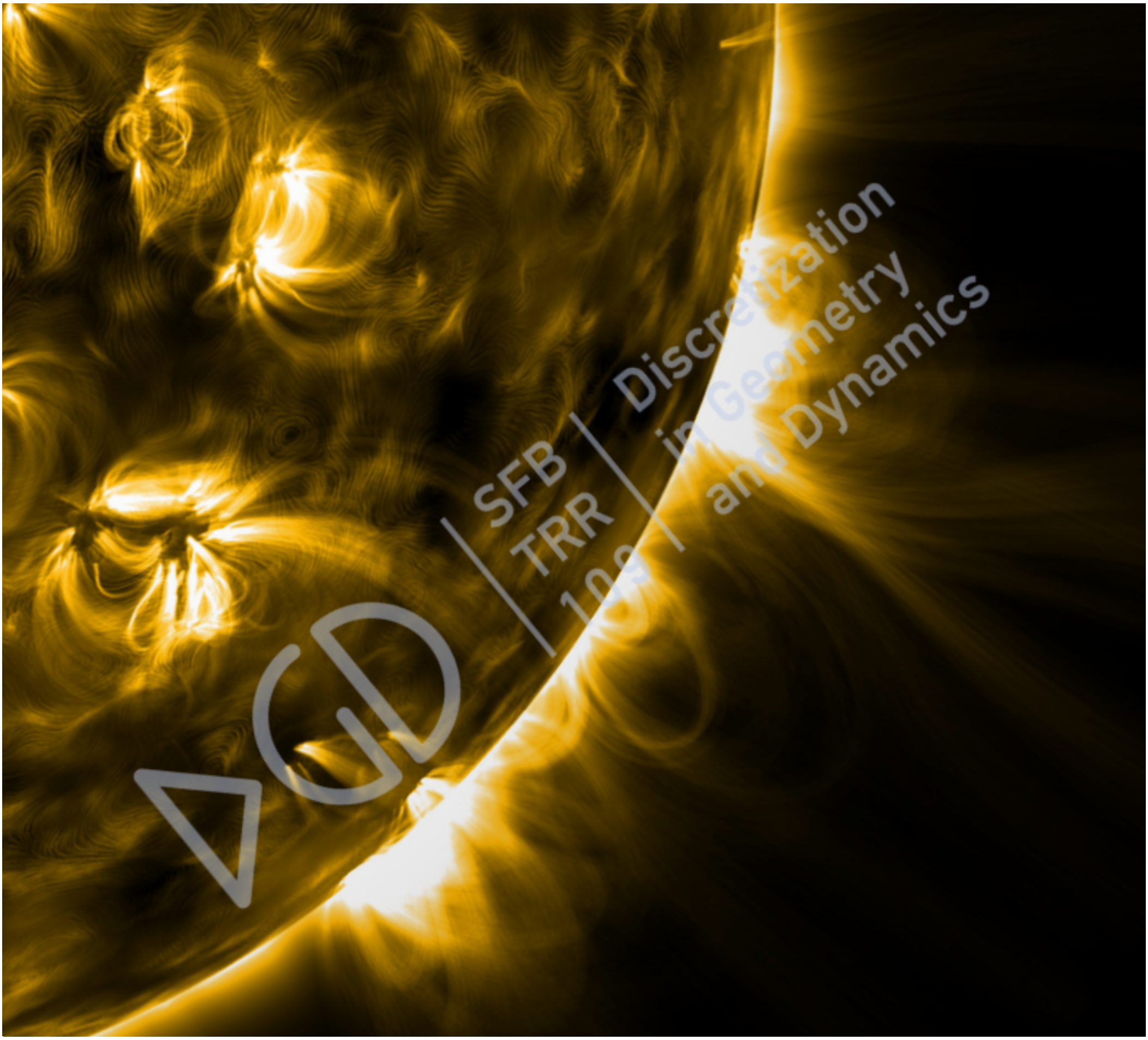

SEPTEMBER

Filament Based Plasma. A procedurally generated visualization of a solar atmosphere. It was computed using a new variational characterization of solutions of the magnetohydrostatic equation. This characterization is inspired by the dominat fibrous structure of the solar corona and based on a Lagrangian curve representation of the atmosphere.

Contributors. *Marcel Padilla, Oliver Gross, Felix Knöppel, Albert Chern, Ulrich Pinkall, Peter Schröder*

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
28	29	30	31	1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
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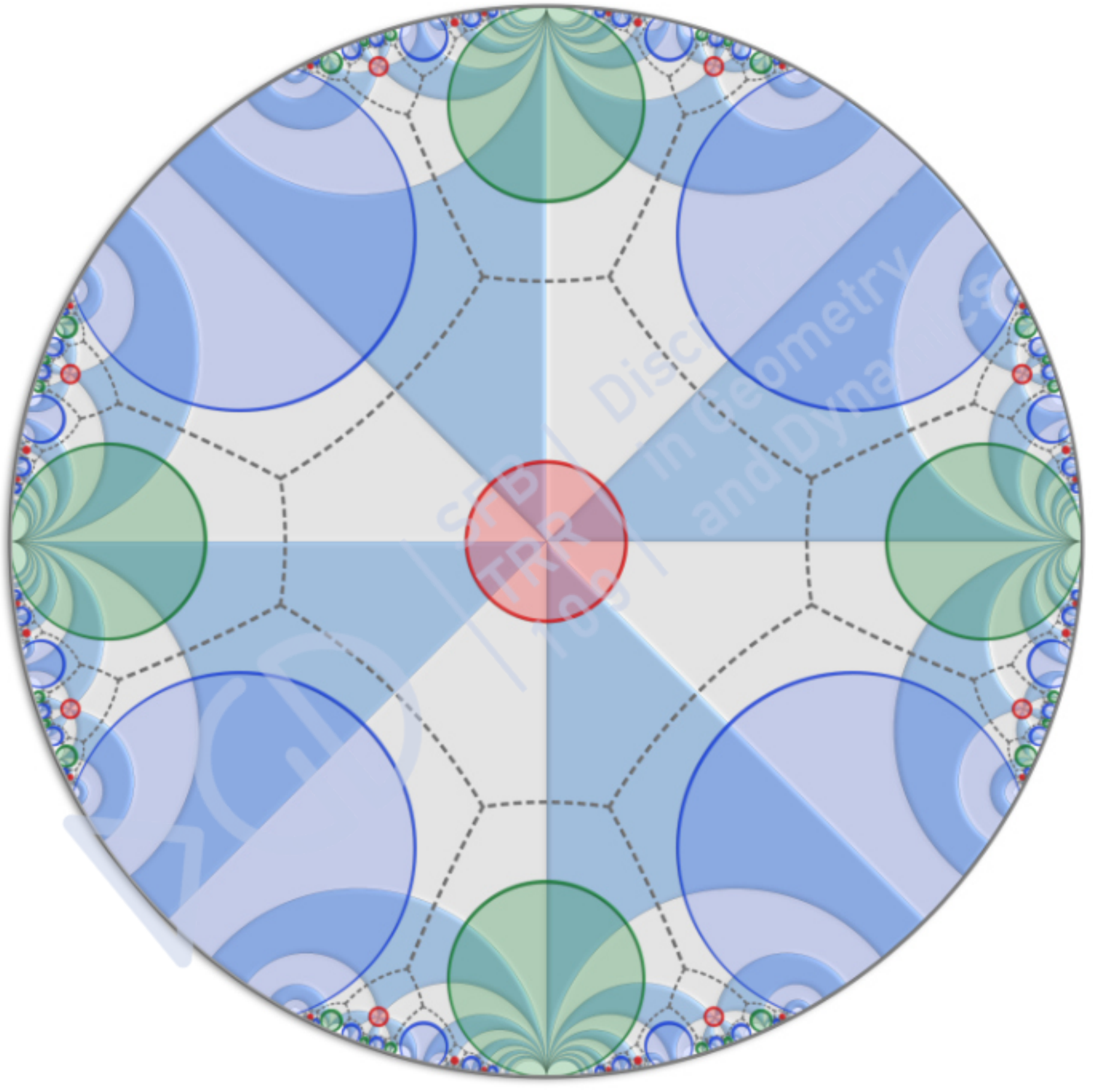

OCTOBER

Canonical Tessellations of Decorated Hyperbolic Surfaces. Shown is a tessellation of the hyperbolic plane corresponding to a weighted Delaunay triangulation (blue-gray chequered) and its dual weighted Voronoi decomposition (dashed lines) of a decorated hyperbolic surface. The surface is obtained as the quotient of the hyperbolic plane by a Fuchsian group. It has a cone-point (red), a cusp (green) and a flare (blue).

Contributor. *Carl O. R. Lutz*

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
25	26	27	28	29	30	1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
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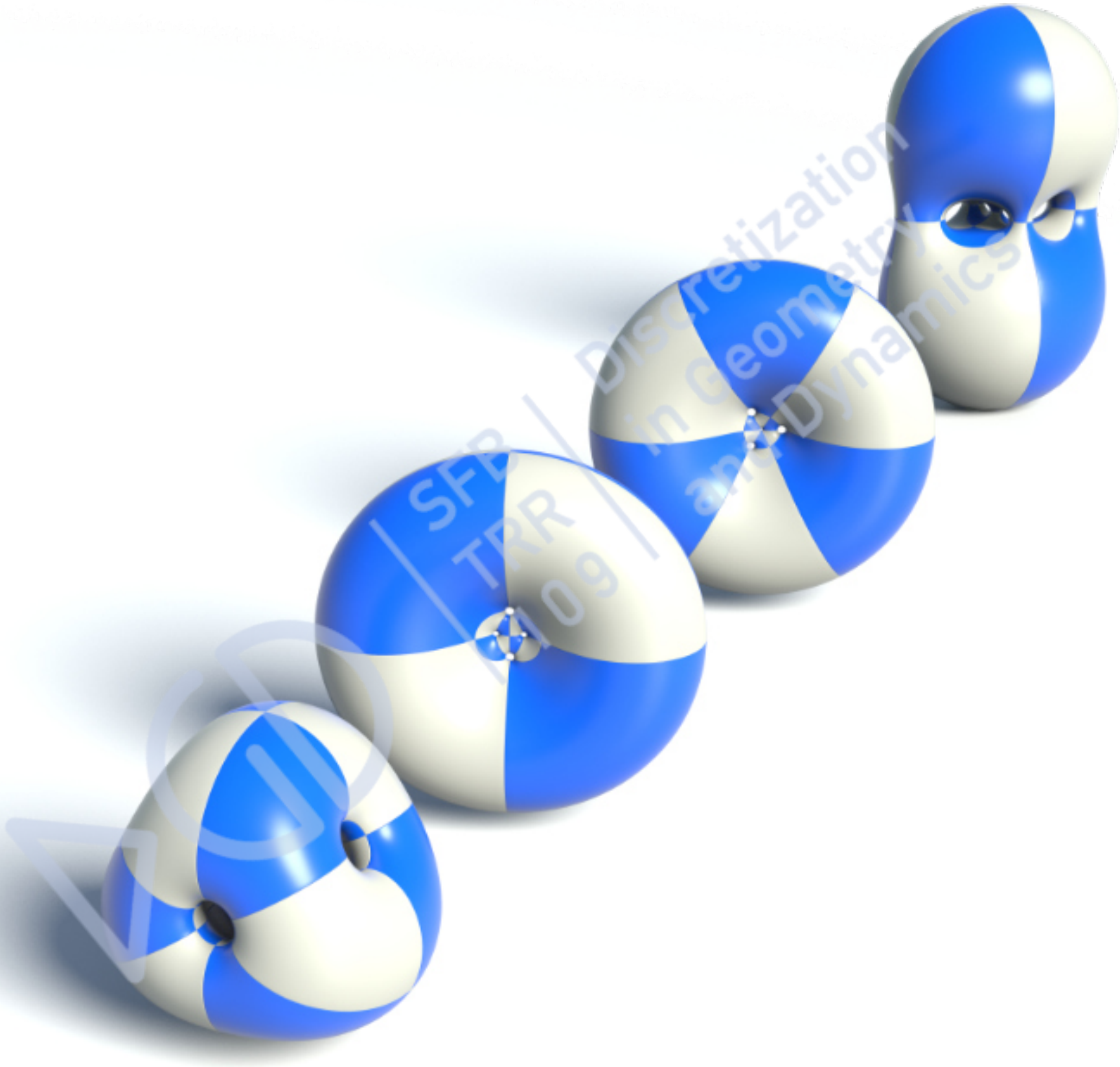

NOVEMBER

Minimal Surfaces in the 3-Sphere. These four surfaces are obtained by stereographically projecting a minimal surface of genus four in the 3-sphere in four different ways. The surface is built by reflecting a fundamental pentagon (blue-white pattern). Each of the projections emphasizes a different symmetry of the surface.

Contributors. *Alexander I. Bobenko, Sebastian Heller, Nicholas Schmitt*

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
30	31	1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
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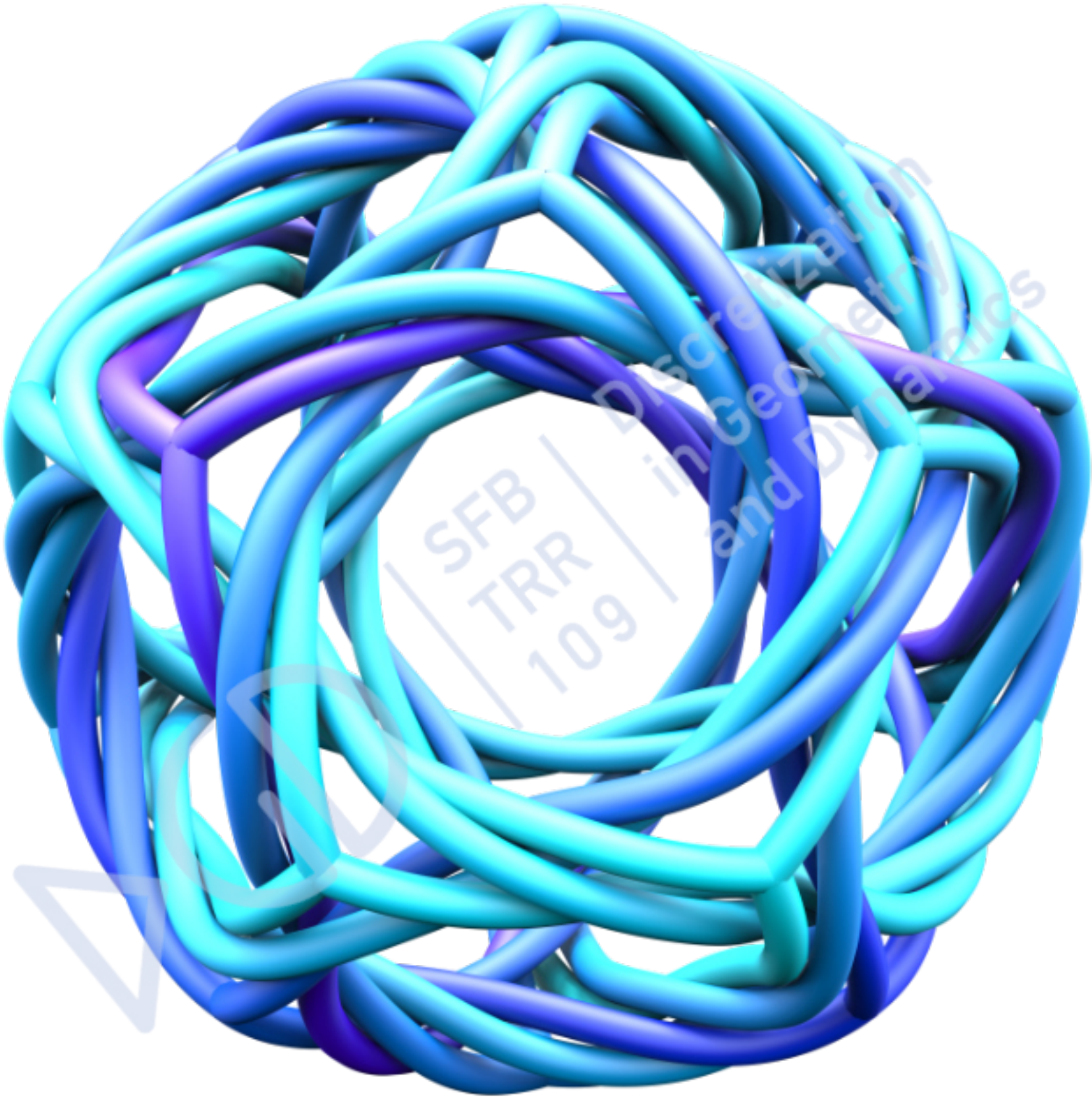

DECEMBER

Entanglement on the Dodecahedron. This structure is the entanglement of ten θ-polyhedra, wound around the edges of a dodecahedron. It has point group symmetry almost as high as the dodecahedron itself.

Contributors. *Stephen T. Hyde, Myfanwy E. Evans*

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
27	28	29	30	1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31
1	2	3	4	5	6	

CONTENTS & REFERENCES

JULY – DECEMBER

July

Isothermic Tori. In this project, the researchers explicitly construct a pair of immersed tori in three dimensional Euclidean space that are related by a mean curvature preserving isometry. This resolves a longstanding problem on whether the metric and mean curvature function determine a unique compact surface. The construction is based on a relationship between Bonnet pairs and isothermic surfaces. In particular, the Bonnet pairs arise from isothermic surfaces with one family of planar curvature lines.

Reference.

Bobenko, A.I., Hoffmann, T., Sageman-Furnas, A.O. (2021). Compact Bonnet Pairs: isometric tori with the same curvatures. arXiv: 2110.06335 [math.DG]

August

Discrete Surfaces with a Constant Ratio of Principal Curvatures. These surfaces are computed through numerical optimization, based on a discrete version of isogonal characteristic nets. The computational model serves as an experimental basis for mathematical studies. The shown image raises the question whether, or under which conditions, combinatorics-preserving deformations of minimal surfaces towards surfaces with a constant principal curvature ratio are possible.

Reference.

Wang, H., Pottmann, H. (2022). Characteristic parametrizations of surfaces with a constant ratio of principal curvatures. *Comp. Aided Geom. Design*, 93

September

Filament Based Plasma. The arcs visible in our sun's atmosphere are some of the most awe-inspiring natural spectacles. Their visual depiction is of great interest in scientific visualization, special effects, and games. In this project we model these prominent features with the help of a new variational characterization of the solutions to the magnetohydrostatics equation. It is based on a structure-preserving discretization of the solar corona.

Reference.

Padilla, M., Gross, O., Knöppel, F., Chern, A., Pinkall, U., Schröder, P. (2022). Filament based plasma. *ACM Trans. Graph.*, 41(4), 153:1 – 153:14

October

Canonical Tessellations of Decorated Hyperbolic Surfaces. A decoration of a hyperbolic surface is a choice of circle, horocycle or hypercycle about each cone point, cusp or flare of the surface, respectively. Each decoration induces a canonical tessellation and dual decomposition of the underlying surface. They exhibit a rich geometric structure, including a characterization using the hyperbolic geometric analogue of Delaunay's empty discs, close connections to convex hulls in Minkowski space and a flip algorithm.

Reference.

Lutz, C.O.R. (2022). Canonical Tessellations of Decorated Hyperbolic Surfaces. arXiv: 2206.13461 [math.GT]

November

Minimal Surfaces in the 3-Sphere. The surface presented in this month was obtained in a project aiming to construct symmetric minimal and constant mean curvature surfaces in spherical, Euclidean and hyperbolic 3-space from fundamental polygons. The construction is based on loop group factorization methods using the theory of integrable systems.

Reference.

Bobenko, A.I., Heller, S., Schmitt, N. (in preparation). New minimal surfaces in the 3-sphere based on fundamental polygons.

December

Entanglement on the Dodecahedron. Tangled tetrahedra, octahedra, cubes, icosahedra, and dodecahedra are generalizations of classical–untangled–Platonic polyhedra. Like the Platonic polyhedra, all vertices, edges, and faces are symmetrically equivalent. However, the edges of tangled polyhedra are curvilinear, or kinked, to allow entanglement, much like warps and wefts in woven fabrics. Related structures have been observed in synthetic materials and clathrin assemblies within cells.

Reference.

Hyde, S.T., Evans, M.E. (2022). Symmetric tangled Platonic polyhedra. *P. Natl. Acad. Sci. U.S.A.*, 119(1):e2110345118

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The central goal of the SFB/Transregio is to pursue research on the discretization of differential geometry and dynamics. In both fields of mathematics, the objects under investigation are usually governed by differential equations. Generally, the term "discretization" refers to any procedure that turns a differential equation into difference equations involving only finitely many variables, whose solutions approximate those of the differential equation.

The common idea of our research in geometry and dynamics is to find and investigate discrete models that exhibit properties and structures characteristic of the corresponding smooth geometric objects and dynamical processes. If we refine the discrete models by decreasing the mesh size they will of course converge in the limit to the conventional description via differential equations. But in addition, the important characteristic qualitative features should be captured even at the discrete level, independent of the continuous limit. The resulting discretizations constitutes a fundamental mathematical theory, which incorporates the classical analog in the continuous limit.

The SFB/Transregio 109 brings together scientists from the fields of geometry, dynamics and applications, to join forces in tackling the numerous problems raised by the challenge of discretizing their respective disciplines.

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Impressum

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