SFB TRR 109 | Discretization in Geometry and Dynamics

Calendar 2021

CONTENT & REFERENCES

JANUARY – JUNE

January

Principal Symmetric Meshes. The isolines of principal symmetric surface parametrizations run symmetrically to the principal directions. Such nets are useful for various applications in the context of fabrication and architectural design. Their discretizations come naturally with a family of spheres, the so-called Meusnier spheres. Controlling their radii and the intersection angles of isolines facilitates tasks such as generating Weingarten surfaces including constant mean curvature surfaces and minimal surfaces.

Reference.

Pellis, D., Wang, H., Killian, M., Rist, F., Pottmann, H., Müller, C. (2020). Principal Symmetric Meshes. *ACM Trans. Graph. (Proc. SIGGRAPH)*, 39(4), 127:1 – 127:15

February

Orthogonal Ring Patterns. An orthogonal ring pattern is a subcomplex of the planar square grid with a pair of concentric circles, a ring, associated to each vertex, so that neighboring rings are orthogonal and diagonally related circles of an elementary quadrilateral touch in a common point. These ring patterns are governed by the same equation as circle patterns, admitting a variational approach, and give a means to interpolate between dual pairs of the latter.

Reference.

Bobenko, A.I., Hoffmann, T., Rörig, T. (2019). Orthogonal ring patterns. arXiv: 1911.07095 [math.CV]

March

Discrete Conformal Maps. A discrete conformal map acts on a piecewise linear surface by multiplying all edge lengths with scale factors associated to the vertices. The functionals characterizing discrete conformal maps are related to the volumes of ideal hyperbolic polyhedra, giving a connection between polyhedral realization and discrete uniformization problems. Since the considered functionals are convex, they present an explicit means to computing discrete conformal maps.

Reference.

Bobenko, A.I., Pinkall, U., Springborn, B. (2015). Discrete conformal maps and ideal hyperbolic polyhedra. *Geom. Topol.*, 19(4), 2155 – 2215

April

Discrete Chebyshev Nets. In this project, the researchers develop an algorithm which finds a seamless Chebyshev net over an arbitrarily given surface. This is achieved by optimizing the integrability of unit-length guiding fields. The guiding fields are represented by polyvectors, which are multiple tangent vectors modulo a certain permutation group incorporating the symmetric nature of the coordinate directions. The integrability of guiding fields is characterized by commutativity of the tangent vectors.

Reference.

Sageman-Furnas, A.O., Chern, A., Ben-Chen, M., Vaxman, A. (2019). Chebyshev nets from commuting PolyVector fields. *ACM Trans. Graph. (ACM SIGGRAPH Asia)*, 38(6), 172:1 – 172:16

May

Periodic CMC Surfaces. New surfaces with constant mean curvature (CMC) are constructed by loop group factorization methods using the theory of integrable systems. This project focuses on periodic CMC surfaces, surfaces with Delaunay ends and on compact minimal surfaces in the three-dimensional sphere.

Reference.

Bobenko, A.I., Heller, S., Schmitt, N. (in progress). New periodic cmc surfaces and surfaces with ends based on fundamental quadrilaterals.

June

Curved Folding. Pleated structures, generated by folding paper along curved creases, exhibit interesting geometry and computational challenges in both the continuous case and its structure-preserving discretization. Their existence was proved physically by M. and E. Demaine's artwork and has recently been proved by L. Alese.

Reference.

Jiang, C., Mundilova, K., Rist, F., Wallner, J., Pottmann, H. (2019). Curve-pleated structures. *ACM Trans. Graph.*, 38(6), 169:1 – 169:13

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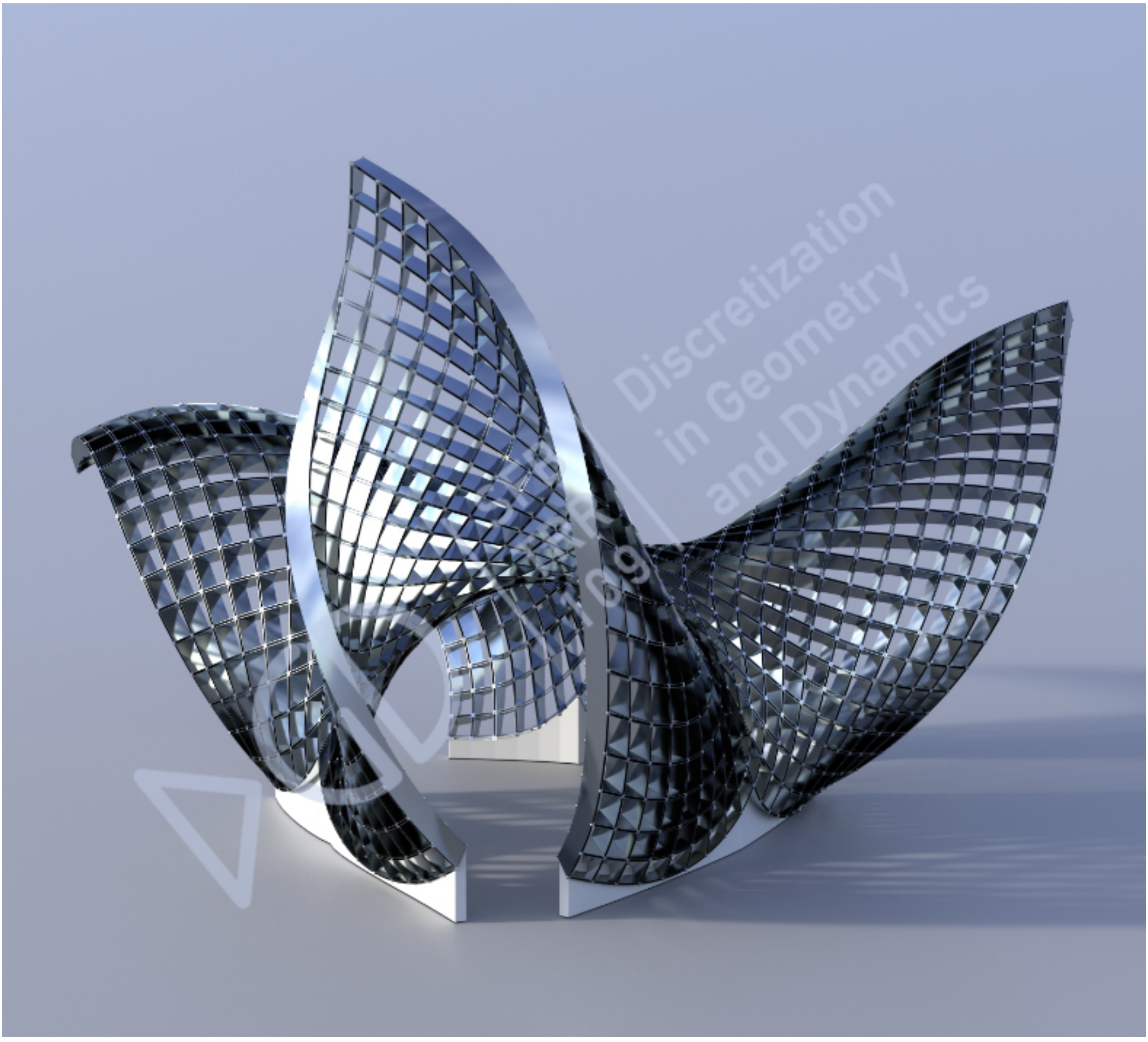

JANUARY

Principal Symmetric Meshes. Rendering of a pavilion, inspired by the work of E. Schling. The underlying surface is a discrete principal symmetric net with constant radius Meusnier spheres, resulting in lamellas sitting orthogonal on the surface and which can be unfolded into parts of circular annuli of constant radius.

Contributors. *Davide Pellis, Hui Wang, Martin Kilian, Florian Rist, Helmut Pottmann, Christian Müller*

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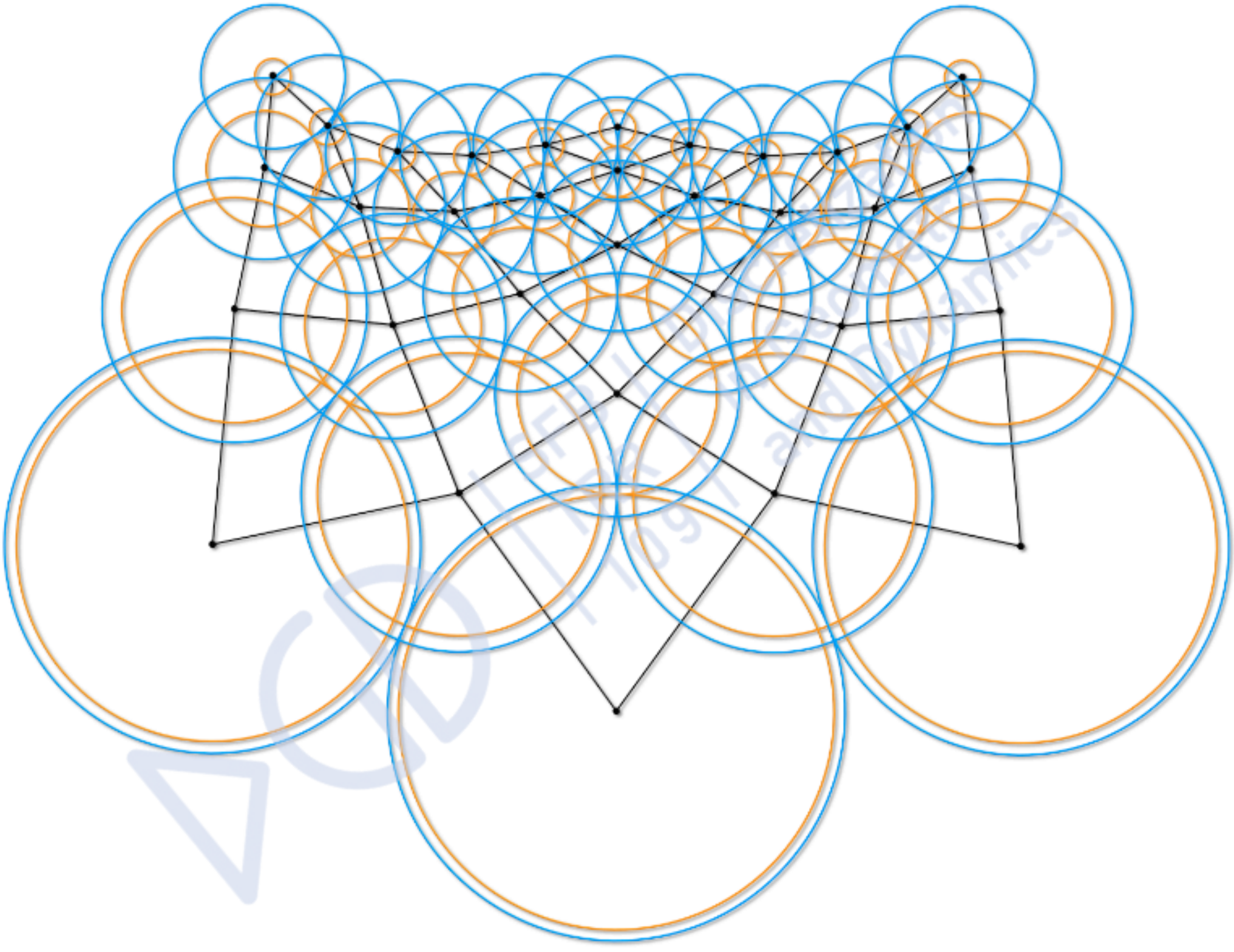

FEBRUARY

Orthogonal Ring Patterns. The image shows an Erf ring pattern belonging to a family of ring patterns converging to Schramm's Erf circle pattern in the limit. Its underlying combinatorics is that of a subcomplex of the planar square grid (black graph). To each vertex a ring is associated, given by a pair of concentric circles (blue and orange circles), such that neighboring rings intersect orthogonally.

Contributors. *Alexander I. Bobenko, Tim Hoffmann, Thilo Rörig*

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MARCH

Discrete Conformal Maps. The photo presents a porcelain teapot manufactured by the Königliche Porzellan-Manufaktur Berlin (KPM). Interpreting the teapot as a torus, ignoring its handle, the line pattern was computed using discrete conformal geometry to map a square grid on a flat torus to the teapot. Note that all lines intersect orthogonally and quadrilaterals are approximated by squares.

Contributors. *Alexander I. Bobenko, Thilo Rörig, Stefan Sechelmann*

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APRIL

Discrete Chebyshev Nets. A Chebyshev net on the Stanford bunny. A Chebyshev net is a quadrilateral mesh (resp. parametrization) where all edge lengths (resp. norm of coordinate vectors) are equal.

Contributors. *Andrew O. Sageman-Furnas, Albert Chern, Mirela Ben-Chen, Amir Vaxman*

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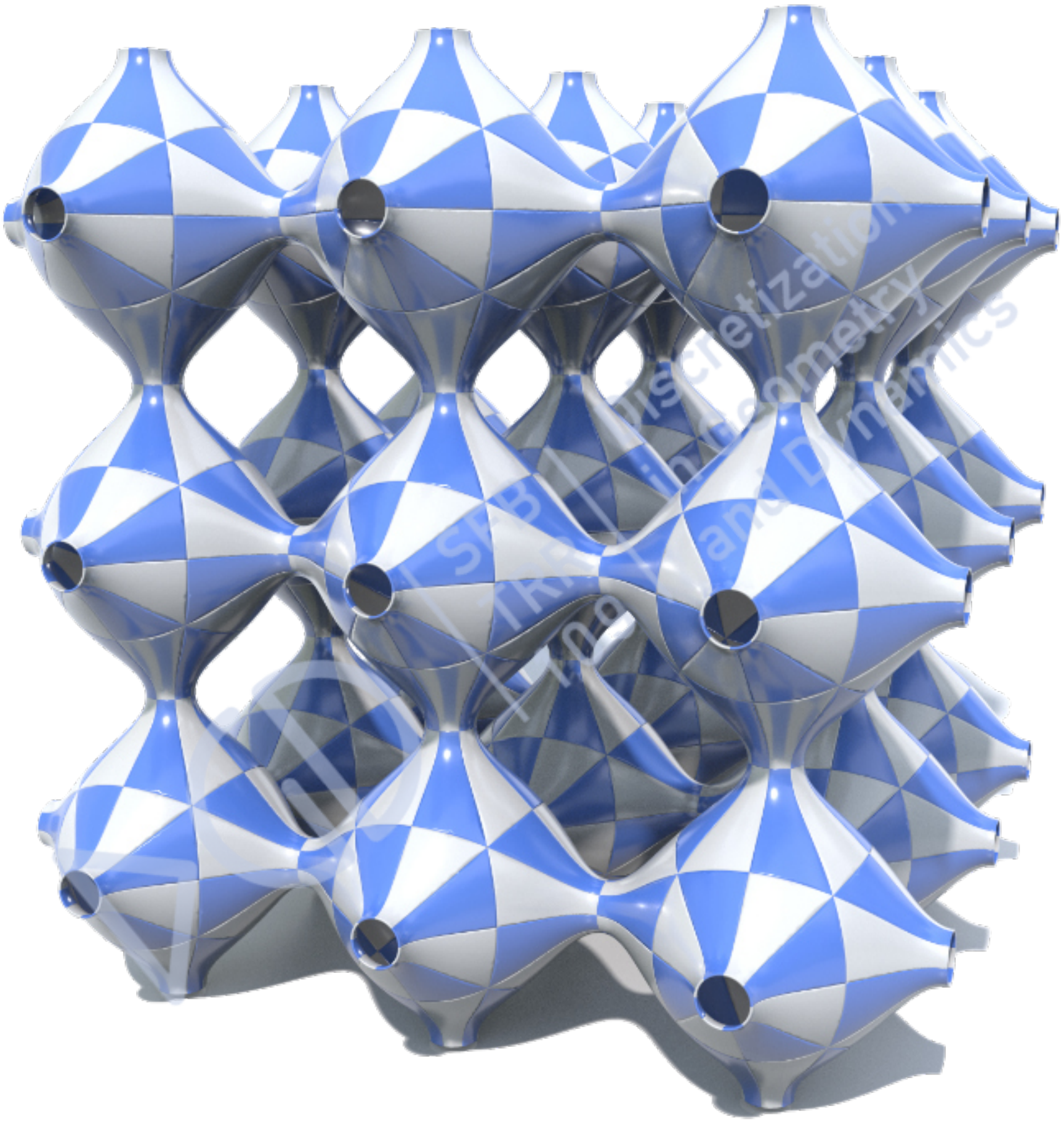

MAY

Periodic CMC Surfaces. This image shows a triply-periodic surface with constant mean curvature. It is constructed by loop group factorization methods using the theory of integrable systems. The surface is separated into congruent quadrilaterals by a pattern of curvature lines.

Contributors. *Alexander I. Bobenko, Sebastian Heller, Nicholas Schmitt*

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JUNE

Curved Folding. This pleated structure is isometric to a planar domain and is smooth except for nested foldlines. It has been computed by propagating pseudo-geodesics from a space curve, followed by optimization. Note that owing to nonzero linking numbers, no continuous unfolding is possible.

Contributors. *Caigui Jiang, Klara Mundilova, Florian Rist, Johannes Wallner, Helmut Pottmann*

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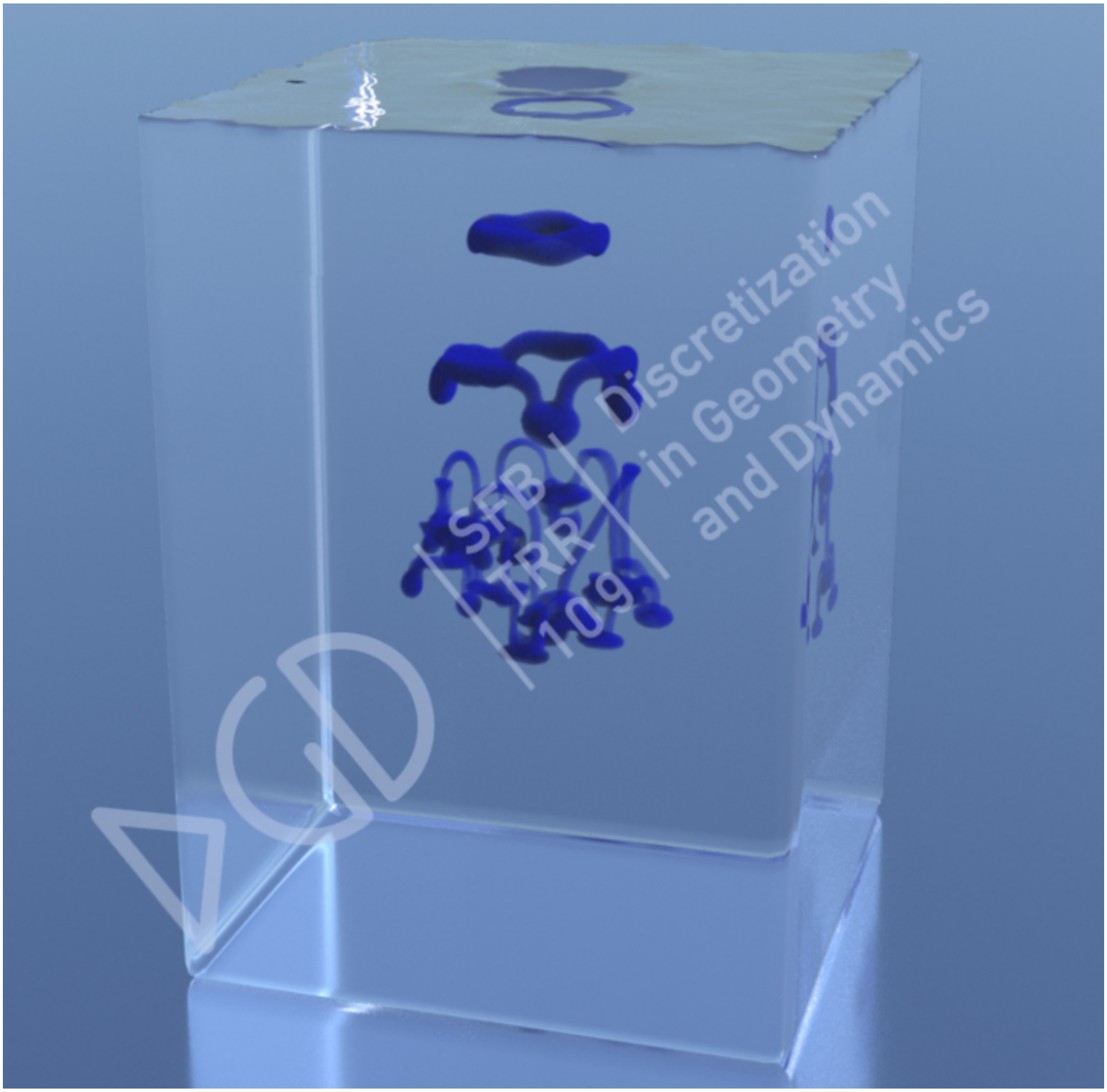

JULY

Ink Chandeliers. This image shows multiple stages of a vortex ring breakdown into a chandelier-like pattern – a phenomenon which is observed when an ink drop is released into water. This simulation was run on a curve-base computation with only a few vertices. The water container around it was just added for visual completion.

Contributors. *Marcel Padilla, Albert Chern, Felix Knöppel, Ulrich Pinkall, Peter Schröder*

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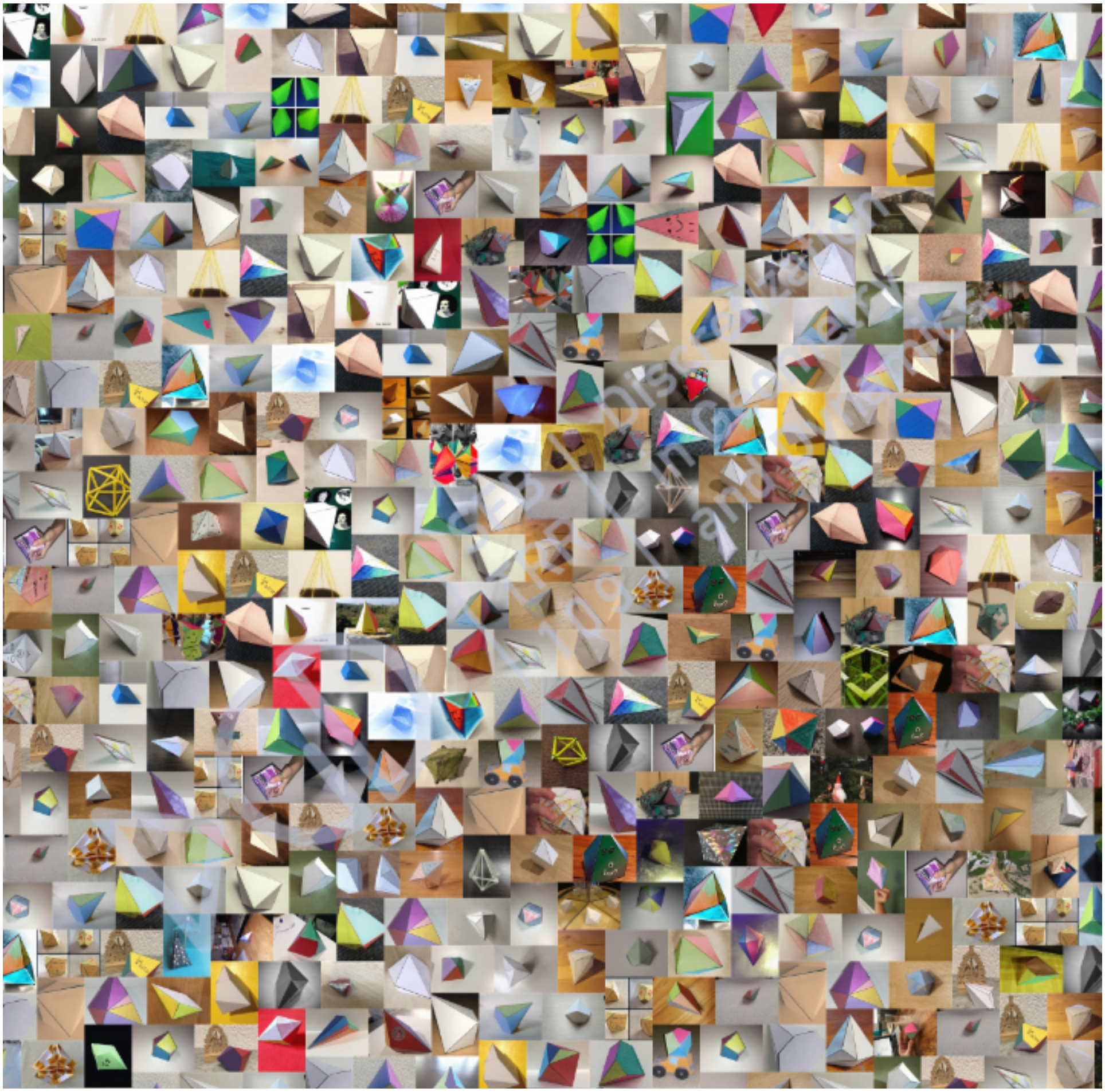

AUGUST

Adopt a Polyhedron. The picture shows a wide collection of the geometric models of polyhedra with up to nine vertices built during the last years. People around the world adopted a polyhedron, gave it a name and a foster home, and realized it in a unique way. On *polytopia.eu* we collect photos of the models and used them to create this mosaic.

Contributors. *Marie-Charlotte Brandenburg, Anna M. Hartkopf, Erin Henning, Mara Kortenkamp, Günther M. Ziegler*

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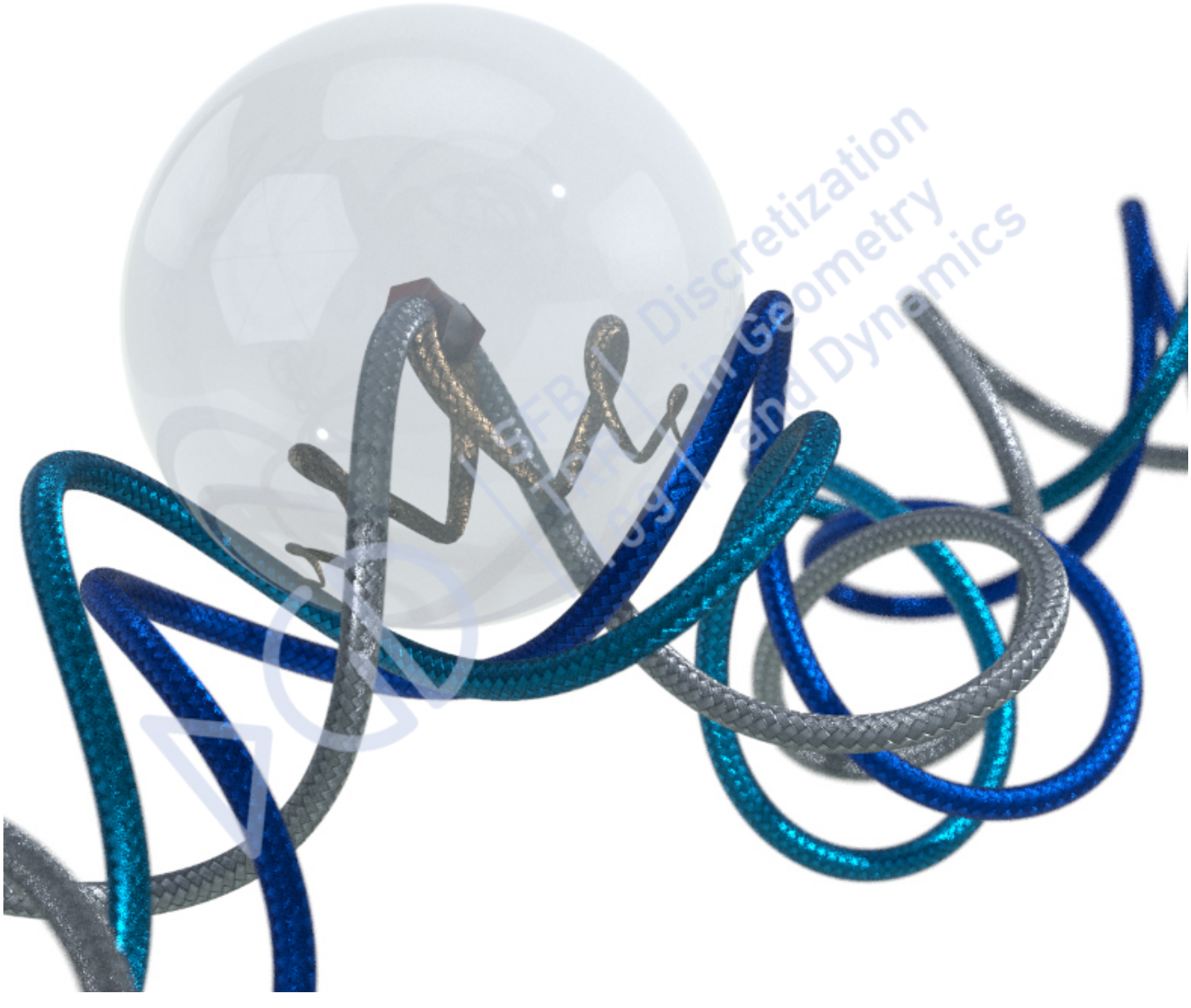

SEPTEMBER

Darboux-Transformation. Given a space curve (silver), a base point on the curve, and a complex spectral parameter (twist and scale), we consider the curve in the associated family (gold) by adding the constant twisting and scaling over the original curve. As the base point moves along the original curve, the limit point of the associated curve traces out a Darboux transform of the original curve.

Contributors. *Albert Chern, Felix Knöppel, Franz Pedit, Ulrich Pinkall*

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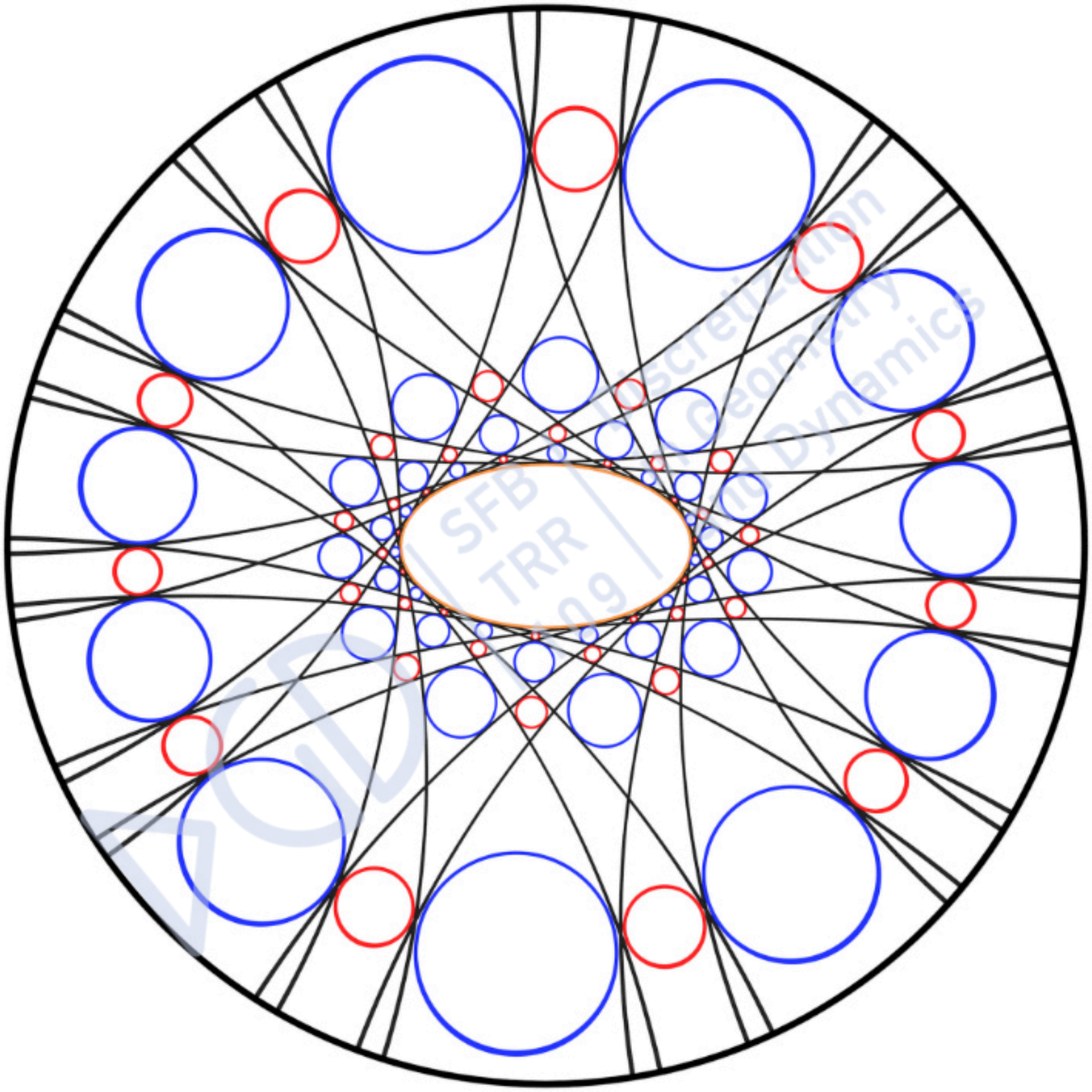

OCTOBER

Checkerboard Incircular Nets. Depicted is a periodic confocal checkerboard incircular net in the Poincaré disk model of the hyperbolic plane. It consists of a family of hyperbolic lines (black arcs) with the combinatorics of the square grid. All lines are tangent to a hyperbolic ellipse (orange curve) and every other quadrilateral is inscribed by a hyperbolic circle (red and blue circles).

Contributors. *Alexander I. Bobenko, Carl O. R. Lutz, Helmut Pottmann, Jan Techter*

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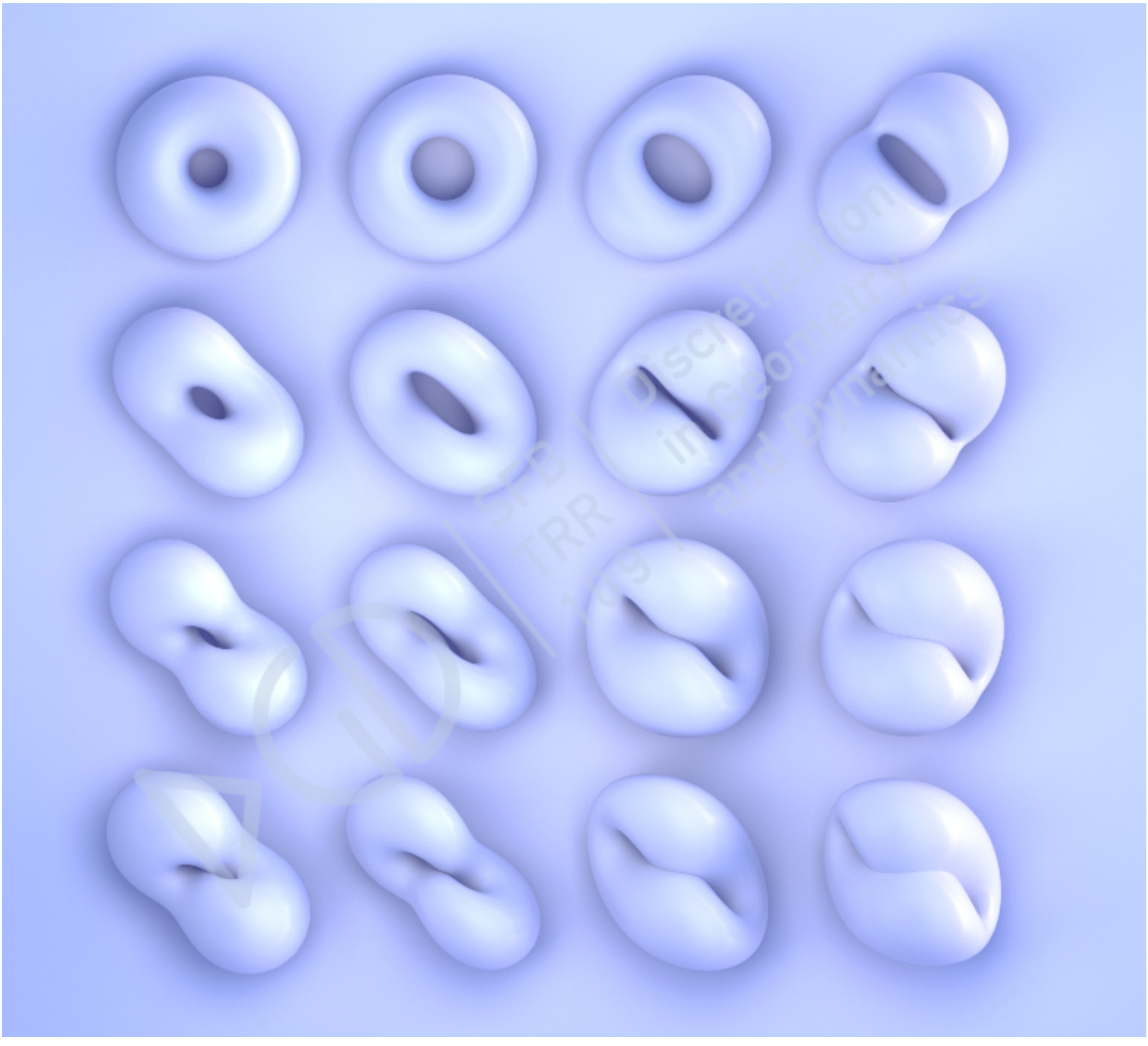

NOVEMBER

Discrete Constrained Wilmore Surfaces. Minimizing the Willmore energy in a fixed conformal class results in geometric realizations of abstract Riemann surfaces in space. Visualized above is a family of discrete conformally constrained Willmore tori parameterized by their intrinsic twists (rows) and aspect ratios (columns).

Contributors. *Yousuf Soliman, Albert Chern, Olga Diamanti, Felix Knöppel, Ulrich Pinkall, Peter Schröder*

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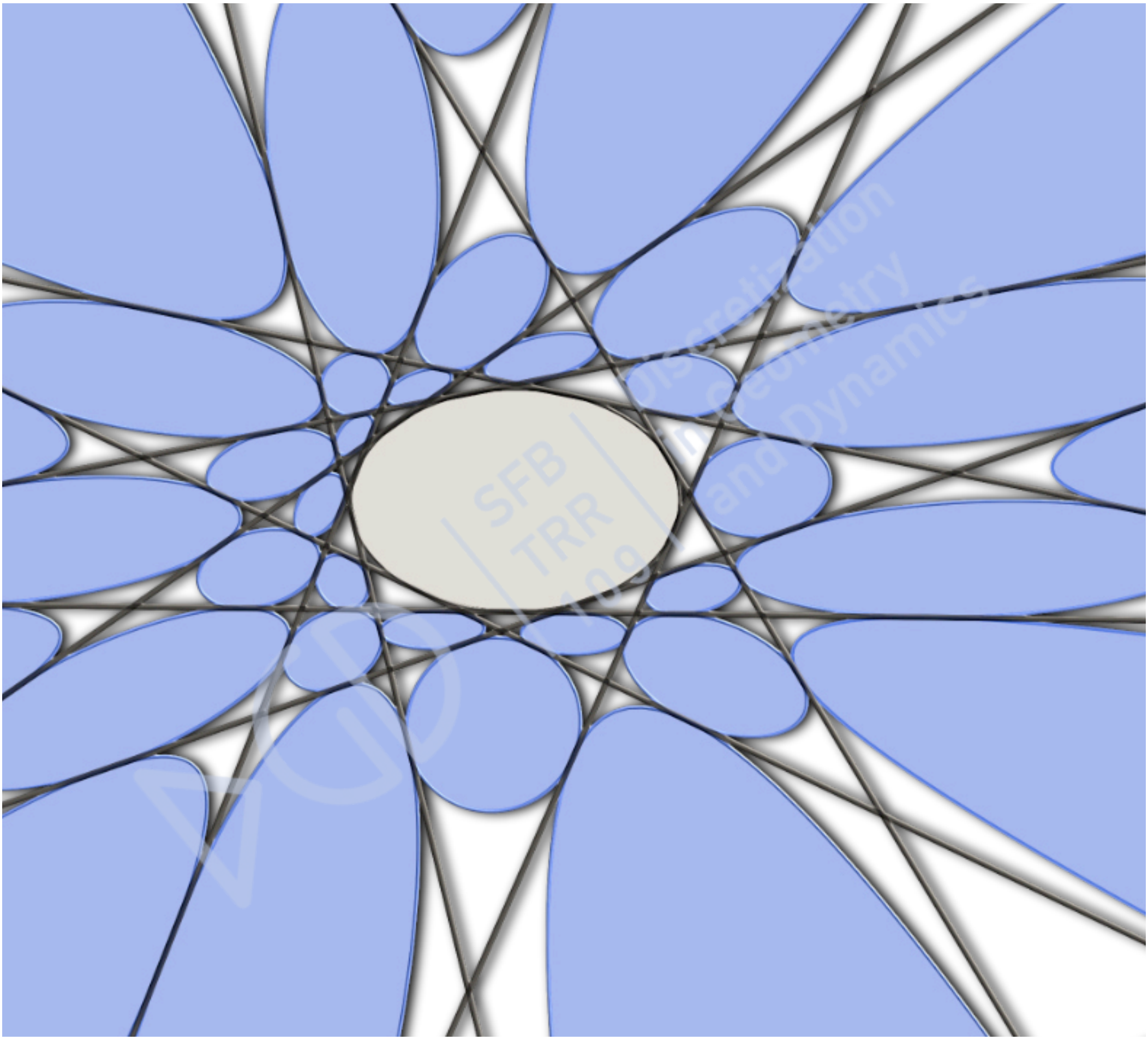

DECEMBER

Nets with Touching Conics. For grids of quadrilaterals with touching inscribed conics (blue), the grid lines are tangent to a conic (gray). The tangency points of the grid lines determine a polygonal chain that is inscribed in the conic. If the polygonal chain satisfies the billiard reflection law, then the grid is related to an incircular net.

Contributors. *Alexander I. Bobenko, Alexander Y. Fairley*

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CONTENT & REFERENCES

JULY – DECEMBER

July

Ink Chandeliers. When a drop of ink is released into water, the flow around the drop transforms it into a vortex ring which tightly captures the ink. By introducing a separate fluid density inside a vortex ring and friction through viscosity, gravity will have notable consequences on the dynamics. Choosing distinctive densities, various physical phenomena can be modeled such as bubble rings and ink chandeliers. A vortex ring can be discretized as a discrete curve allowing for high computational efficiency.

Reference.

Padilla, M., Chern, A., Knöppel, F., Pinkall, U., Schröder, P. (2019). On bubble rings and ink chandeliers. *ACM Trans. Graph.*, 38(4), 129:1 – 129:14

August

Adopt a Polyhedron. Models and their creation were, and still are, a central mathematical discipline but have taken the backseat behind digital visualization over the last decades. With the science communication project “Adopt a Polyhedron” we aim to create an invitation to the general public to actively participate in mathematics. The hands-on approach of actually creating a physical object enables a wide audience to grasp mathematics and geometry beyond abstract understanding.

Reference.

Hartkopf, A.M. (2019). Citizen Art – Collective Mathematical Art to Raise the Public Awareness of Mathematics. *Proc. of Bridges 2018: Math., Art, Music, Arch., Edu., Cult.*, Phoenix, Arizona, Tesselations Publishing, 579 – 584

September

Darboux-Transformation. In this project, the researchers study the hierarchy of integrable dynamical systems on space curves containing the the Da Rios vortex filament flow and the modified Korteweg–de Vries (mKdV) flow. Instead of using the classical Hasimoto transform, the integrable system is described directly in terms of a symplectic structure on the space of curves, an explicit sequence of commuting Hamiltonians, and a notion of associated family of the curve geometry.

Reference.

Chern, A., Knöppel, F., Pedit, F., Pinkall, U. (2018). Commuting Hamiltonian flows of curves in real space forms. arXiv:1809.01394 [math.DG]

October

Checkerboard Incircular Nets. A checkerboard incircular net is a two-parameter family of straight lines with the combinatorics of the square grid such that every other elementary quadrilateral admits an incircle. It is possible to extend this notion past the Euclidean case. A unified treatment in all space forms can be achieved by utilizing Non-Euclidean Laguerre geometry, showing the rich geometric structure and close connection to (discrete) confocal conics of these nets.

Reference.

Bobenko, A.I., Lutz, C.O.R., Pottmann, H., Techter, J. (2020). Non-Euclidean Laguerre geometry and incircular nets. arXiv: 2009.00978 [math.MG]

November

Discrete Constrained Willmore Surfaces. This research project focuses on the relationship between the intrinsic geometry of discrete conformal maps and the extrinsic geometry of discrete surfaces. Variational problems for discrete conformal immersions have many similarities with their smooth counterparts: for example, holomorphic quadratic differentials arise as conformal Lagrange multipliers and isothermic surfaces are precisely the degenerate points of the constraint.

Reference.

Chern, A., Knöppel, F., Pedit, F., Pinkall, U., Schröder, P. (2019). Finding Conformal and Isometric Immersions of Surfaces. arXiv: 1901.09432 [math.DG]

December

Nets with Touching Conics. This research project focuses on nets of quadrilaterals with inscribed touching conics. That is, conics of neighboring quadrilaterals share the same touching point on their common edge line. In particular, families of straight lines with the combinatorics of the square grid building nets with touching conics are investigated. They are related to elliptic billiards.

Reference.

Bobenko, A.I., Fairley, A.Y.(2019). Nets of lines with the combinatorics of the square grid and with touching inscribed conics. arXiv: 1911.08477 [math.AG]

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The central goal of the SFB/Transregio is to pursue research on the discretization of differential geometry and dynamics. In both fields of mathematics, the objects under investigation are usually governed by differential equations. Generally, the term "discretization" refers to any procedure that turns a differential equation into difference equations involving only finitely many variables, whose solutions approximate those of the differential equation.

The common idea of our research in geometry and dynamics is to find and investigate discrete models that exhibit properties and structures characteristic of the corresponding smooth geometric objects and dynamical processes. If we refine the discrete models by decreasing the mesh size they will of course converge in the limit to the conventional description via differential equations. But in addition, the important characteristic qualitative features should be captured even at the discrete level, independent of the continuous limit. The resulting discretizations constitute a fundamental mathematical theory, which incorporates the classical analog in the continuous limit.

The SFB/Transregio 109 brings together scientists from the fields of geometry, dynamics and applications, to join forces in tackling the numerous problems raised by the challenge of discretizing their respective disciplines.

www.discretization.de

Impressum

Collaborative Research Centre/Transregio 109
Discretization in Geometry and Dynamics

Coordinator

Prof. Dr. Alexander I. Bobenko

Technische Universität Berlin

Institut für Mathematik, MA 8–4,

Straße des 17. Juni 136, 10623 Berlin

Editors

Oliver Gross, Carl O. R. Lutz

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