DGD | SFB TRR 109 | Discretization in Geometry and Dynamics

Calendar 2024

CONTENTS & REFERENCES

JANUARY – JUNE

January

Deployable strip structures. The new concept of a C-mesh captures kinetic structures deployable from a flat collapsed state. Surprisingly, quadrilateral C-meshes enjoying a straight collapse correspond to asymptotic Chebyshev nets on K-surfaces, while a circular collapsed state corresponds to Linear-Weingarten surfaces. More general collapses actually lead to a smaller class of shapes. We provide tools for design, and an application to paneling architectural skins with spherical panels of constant radius.

Reference.

Liu, D., Pellis, D., Chiang, Y.-C., Rist, F., Wallner, J., Pottmann, H. (2023). Deployable strip structures. *ACM Trans. Graph.*, 42(4), 103:1 – 103:16

February

Rolling Spheres and the Willmore Energy. Studying discrete surfaces in the conformal 3-sphere provides a fruitful ground to discover smooth-to-discrete correspondences in differential geometry. This research project focuses on understanding the Möbius invariant discretization of the Willmore energy. It can be understood geometrically as the curvature of a connection obtained by rolling the circumspheres around a vertex, mirroring the smooth theory where the circumspheres are replaced by mean curvature spheres.

Reference.

Knöppel, F., Pinkall, U., Schröder, P., Soliman, Y. (2023) Rolling spheres and the Willmore energy. *Preprint*, arXiv: 2311.02241 [math.DG]

March

Tractrix Surfaces. Cone-nets are networks of two families of curves parametrizing a surface such that along each curve of one family there exists a cone in tangential contact with the surface. A cone-net is a conjugate net, projectively invariant and it is characterized by the existence of transformations governed by a system of PDEs. Special cone-nets are smooth and discrete tractrix surfaces which are principal cone-nets with constant geodesic curvature along one family of parameter curves.

Reference.

Kilian, M., Müller, C., Tervooren, J. (2023) Smooth and Discrete Cone-Nets. *Results Math.*, 78, 110

April

Plasma Knots. Imagine an electrically-conducting fluid initially at rest. If we allow the fluid to relax to some minimum-energy state subject to the ideal magnetohydrodynamic equations, can we understand the relaxation process, or predict the relaxed state? This is the essence of the “magnetic relaxation” problem. This project employs geometric tools to tackle topology-preserving energy minimization in fluid dynamics.

Reference.

Gross, O., Pinkall, U., Schröder, P. (2023) Plasma Knots. *Physics Letters A*, 480, 128986

May

Triangle Mesh with Spherical Faces. Sphere meshes with spherical faces and circular arcs as edges constitute a surface class which is invariant under Möbius transformations acting on all of its elements. Important for applications is the existence of a geometric support structure formed by circular annuli offsetting the edges, whose carrier planes around a vertex pass through an axis. All such meshes with triangular combinatorics are characterized in terms of non-Euclidean geometries.

Reference.

Kilian, M., Ramos Cisneros, A.S., Müller, C., Pottmann, H. (2023) Meshes with Spherical Faces. *ACM Trans Graph.*, 42(6), 177:1 – 177:19

June

Solving the Bonnet Problem. The documentary “Solving the Bonnet Problem” follows the work of three mathematicians as they collaborate to solve a long-standing geometrical problem proposed by Pierre Ossian-Bonnet. Alongside their work, the film explores the lives and contributions of Bonnet and his contemporary Gaston Darboux, as well as the history of French mathematics in the 19th century. Captivating computer graphics help unravel intricate concepts and make them accessible to all.

Reference.

www.discretization.de/movies

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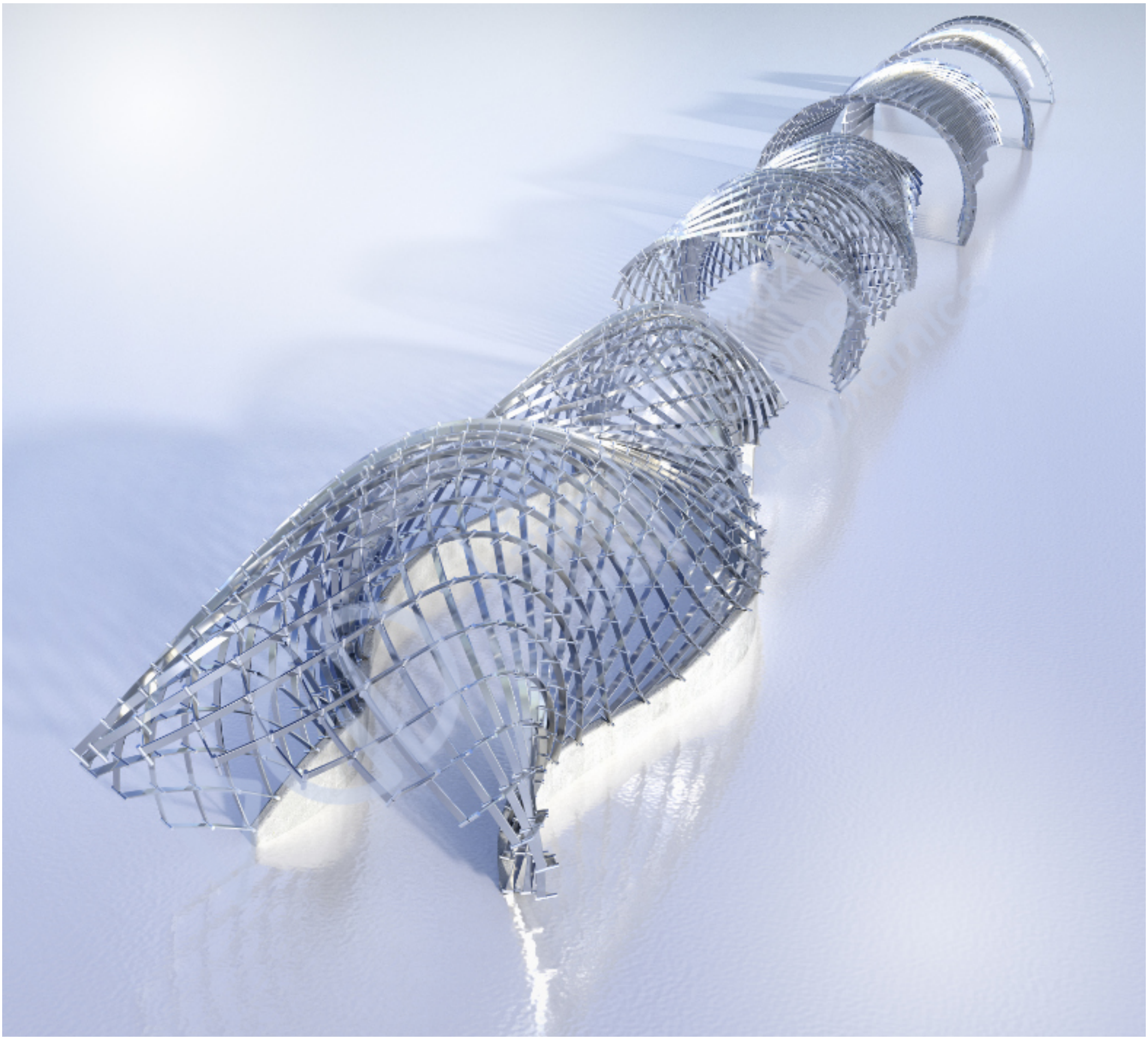

JANUARY

Deployable Strip Structures. Linear-Weingarten surfaces of hyperbolic type are precisely the shapes achievable as square grid arrangements of strips that deploy from a flat state. The deployment discretizes isometric deformation of a Chebyshev net of curves of constant normal curvature.

Contributors. *Daoming Liu, Davide Pellis, Yu-Chou Chiang, Florian Rist, Johannes Wallner, Helmut Pottmann*

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FEBRUARY

Rolling Spheres and the Willmore Energy. Rolling mean curvature spheres over a surface can be used to measure the Willmore energy, which plays a central role in the geometry of surfaces in the conformal 3-sphere. Minimization of this energy under conformality constraints realizes high genus abstract Riemann surfaces in geometrically optimal ways in the 3-sphere.

Contributors. *Felix Knöppel, Ulrich Pinkall, Peter Schröder, Yousuf Soliman*

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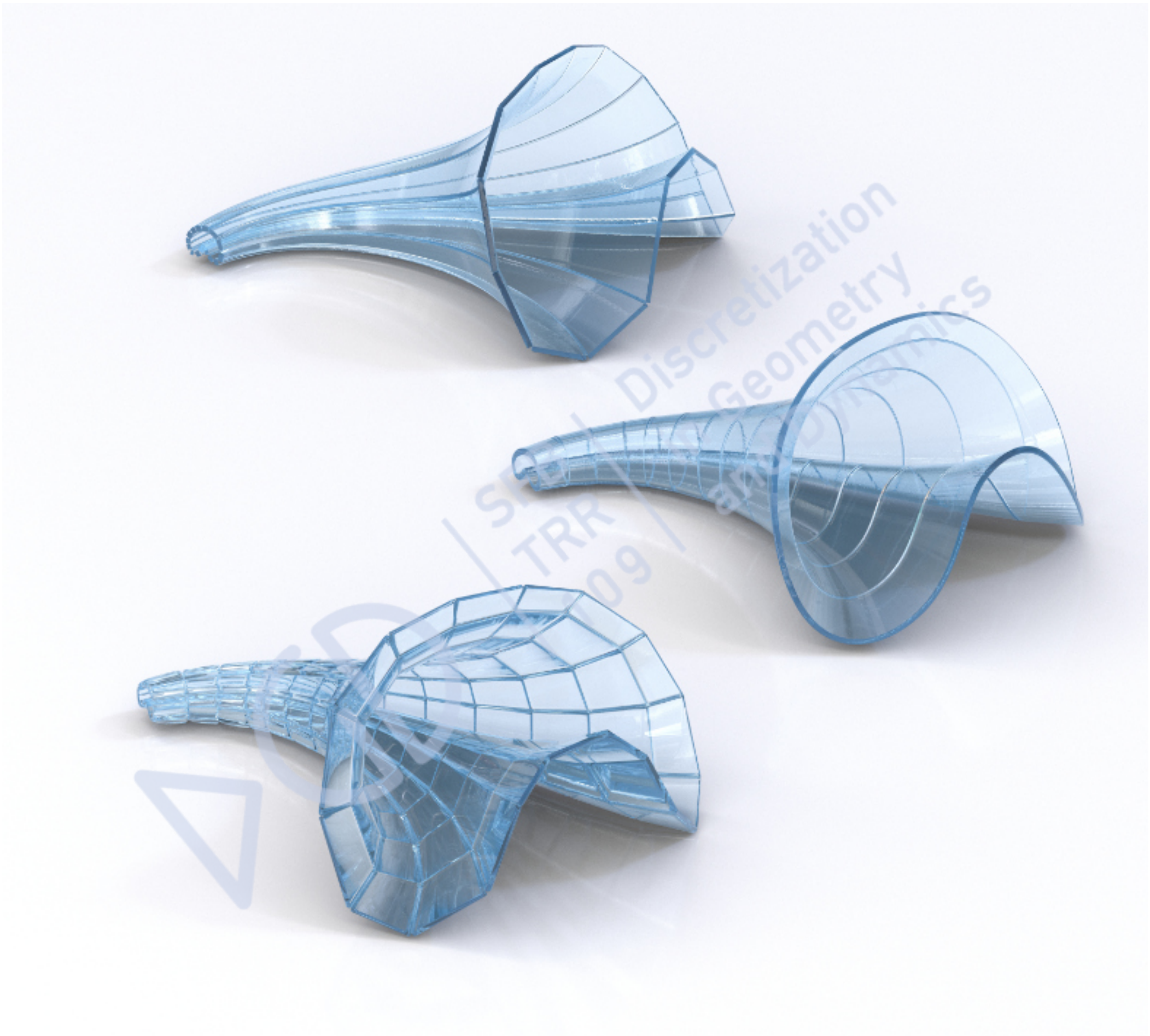

MARCH

Tractrix Surfaces. A cone-net is a parametrization where cones are in tangential contact with the surface along each curve of one family of parameter lines. By pulling spherical curves along space curves we obtain tractrix surfaces as examples of such cone-nets. We see the same shape fully discrete (front) and semi-discrete (center and back).

Contributors. *Martin Kilian, Christian Müller, Jonas Tervooren*

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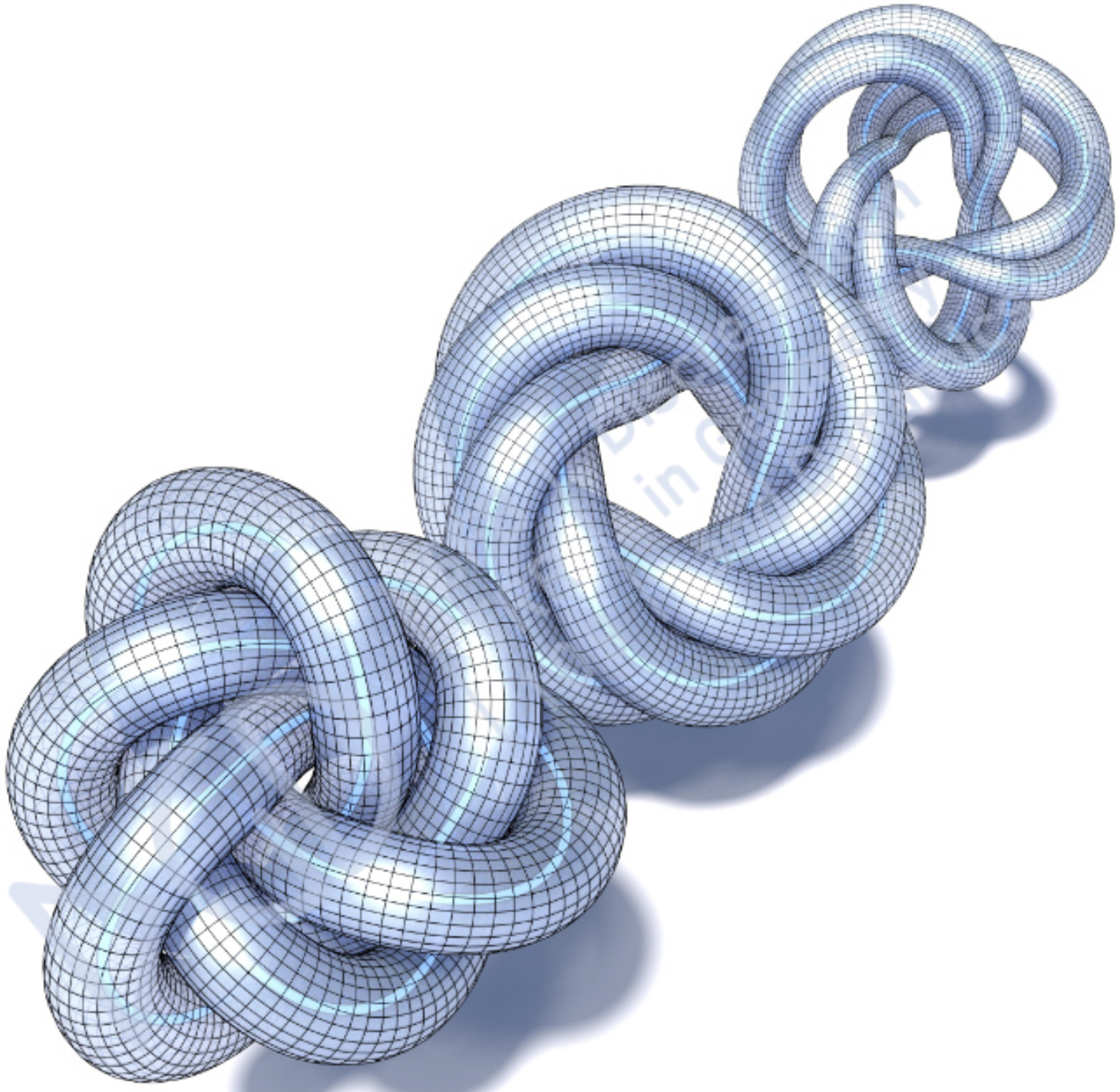

APRIL

Plasma Knots. The time-evolution of a link consisting of two knots during the process of so-called “ideal magnetic relaxation”. The curves with thickness can be understood as discretization of bundles of magnetic field lines in an ideal plasma and the process itself can be reduced to a geometry optimization problem.

Contributors. *Oliver Gross, Ulrich Pinkall, Peter Schröder*

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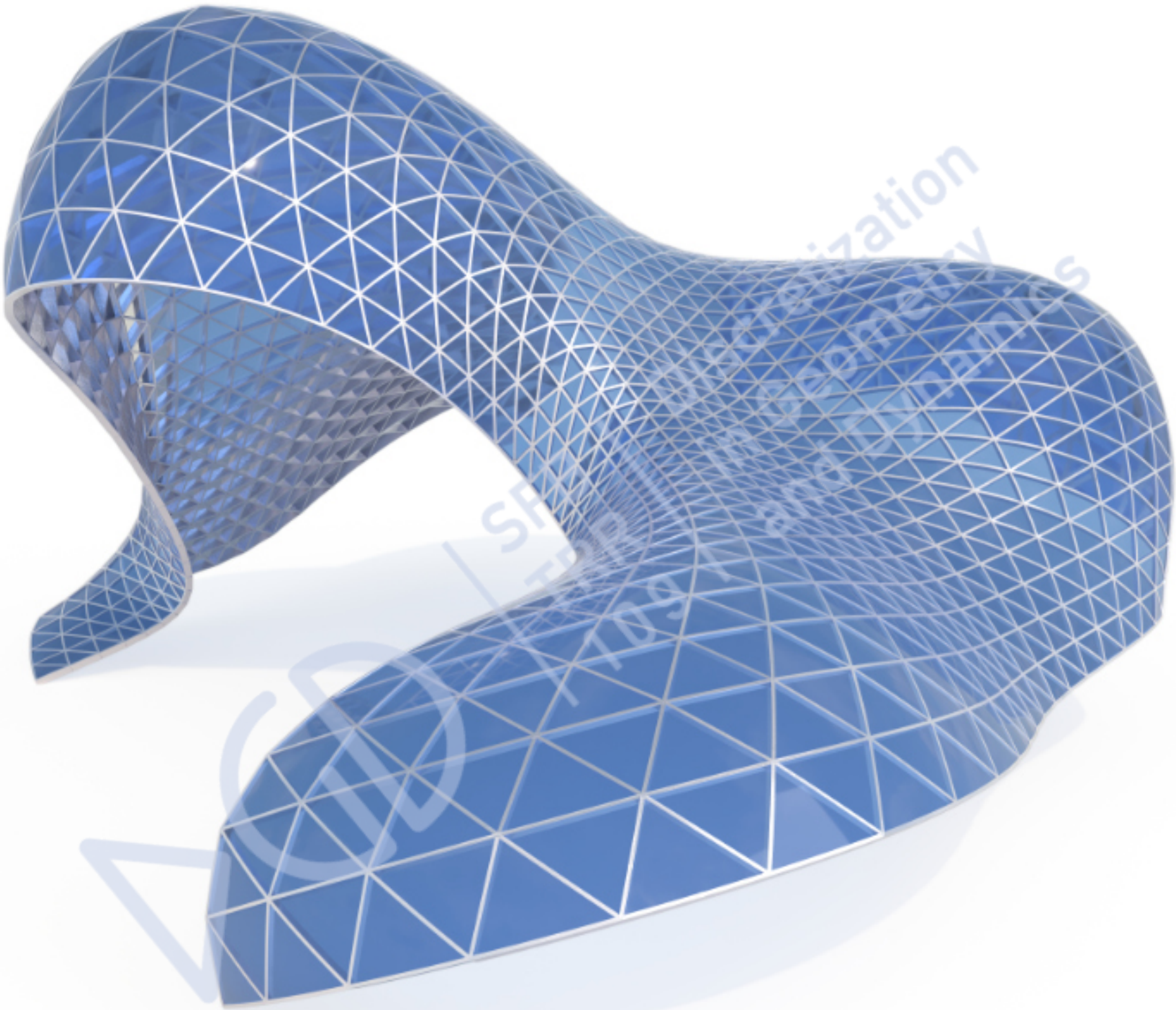

MAY

Triangle Mesh with Spherical Faces. A triangle mesh with spherical patches as faces and circular arcs as edges is a truly Möbius invariant mesh. Around each vertex the planes of the circular edges meet in an axis. This is only possible if all face-spheres are orthogonal to a common sphere. It is therefore a polyhedral surface in hyperbolic space.

Contributors. *Martin Kilian, Anthony S. Ramos Cisneros, Christian Müller, Helmut Pottmann*

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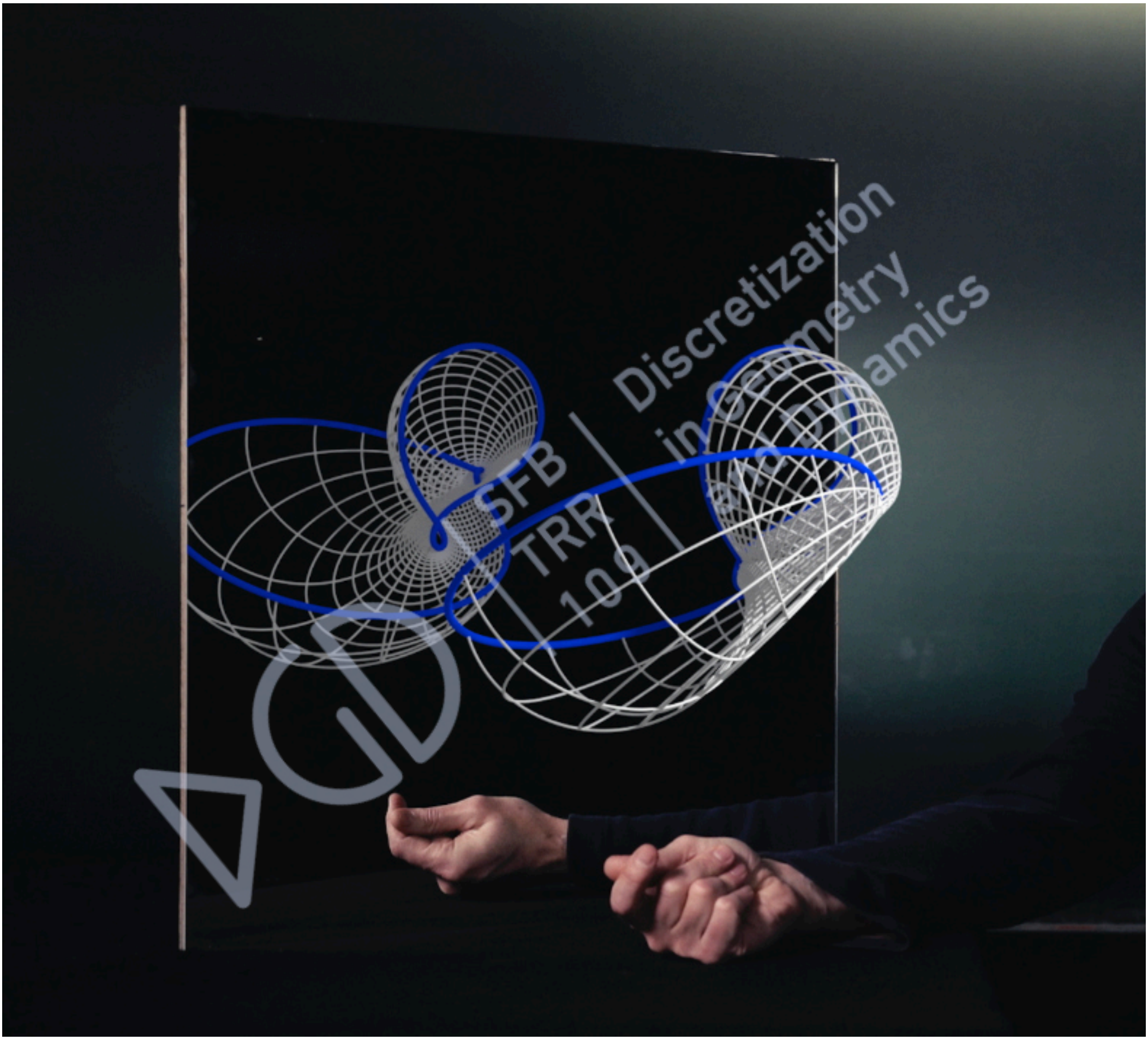

JUNE

Solving the Bonnet Problem. A freeze frame illustrating the concept of fundamental pieces of surfaces from the documentary “Solving the Bonnet problem” by Ekaterina Eremenko. The documentary invites you on the journey of solving a complex mathematical problem. You can witness some of the missteps, breakthroughs, and successes during this process.

Contributors. *Alexander I. Bobenko, Ekaterina Eremenko, Giulio Rasi, Théo Tyburn, Sara Samy*

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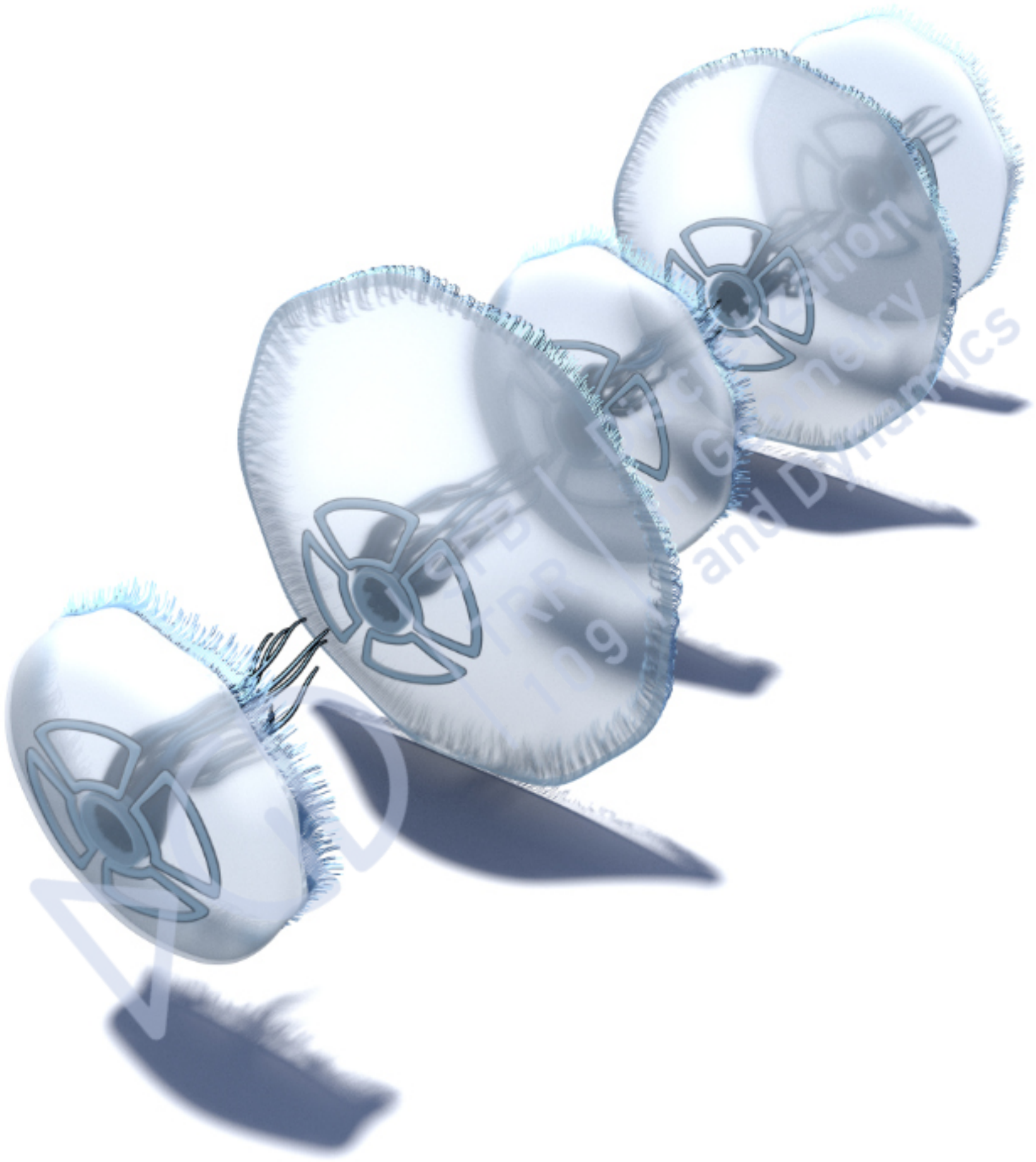

JULY

Motion from Shape Change. Freeze frames that capture the graceful locomotion of a swimming jellyfish. Its translational motion is driven by the principle of minimizing the total energy dissipation to the surrounding medium, resulting from the rhythmic undulation of its umbrella. Many movements in the animal kingdom can be mapped in this way.

Contributors. *Oliver Gross, Yousuf Soliman, Marcel Padilla, Felix Knöppel, Ulrich Pinkall, Peter Schröder*

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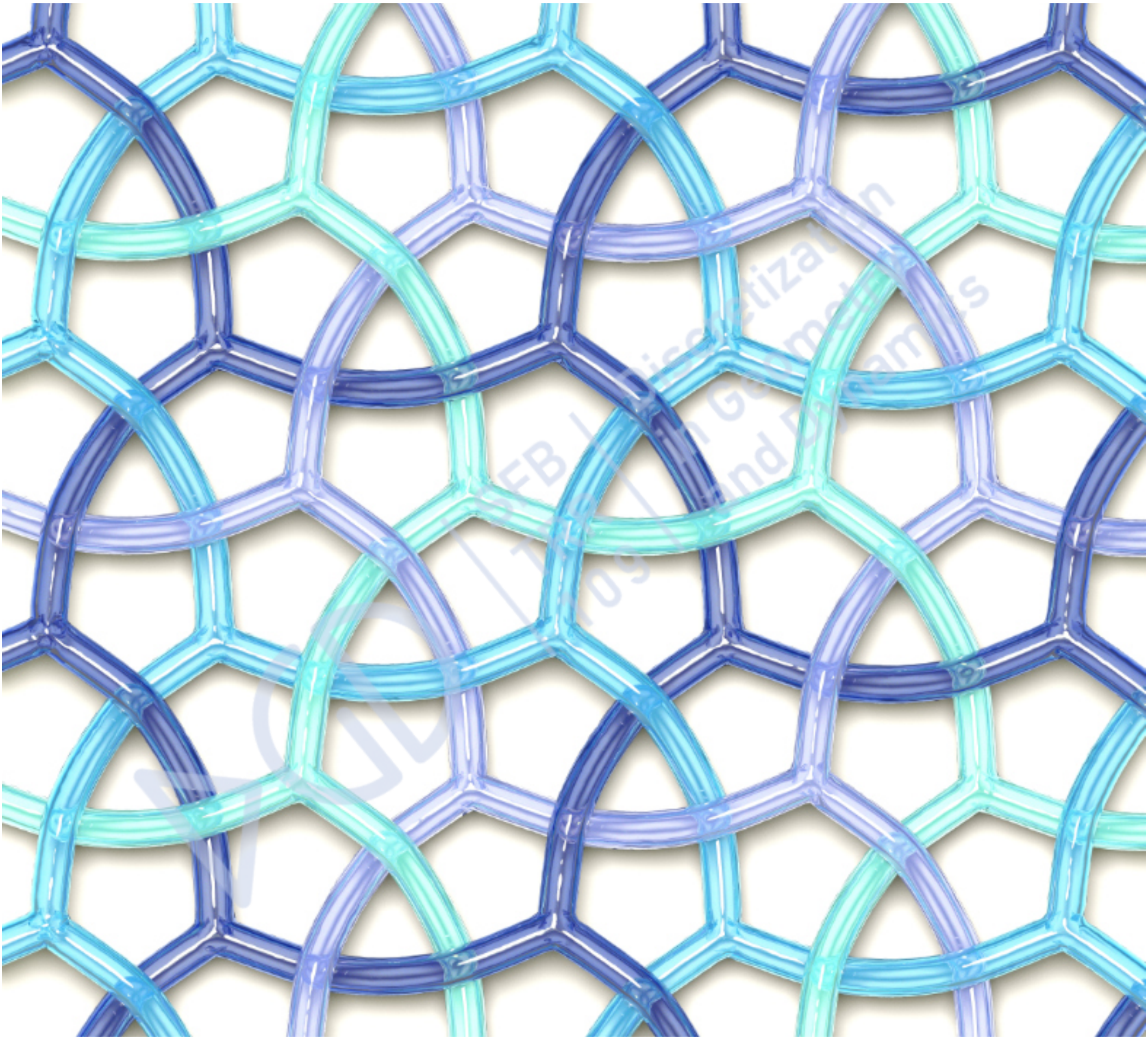

AUGUST

Symmetric Tangling of Honeycomb Networks. The weaving of four regular honeycomb networks into a symmetric pattern. The enumeration and characterisation of such entangled networks supports the understanding of entanglement in chemical frameworks.

Contributors. *Myfanwy Evans, Stephen Hyde*

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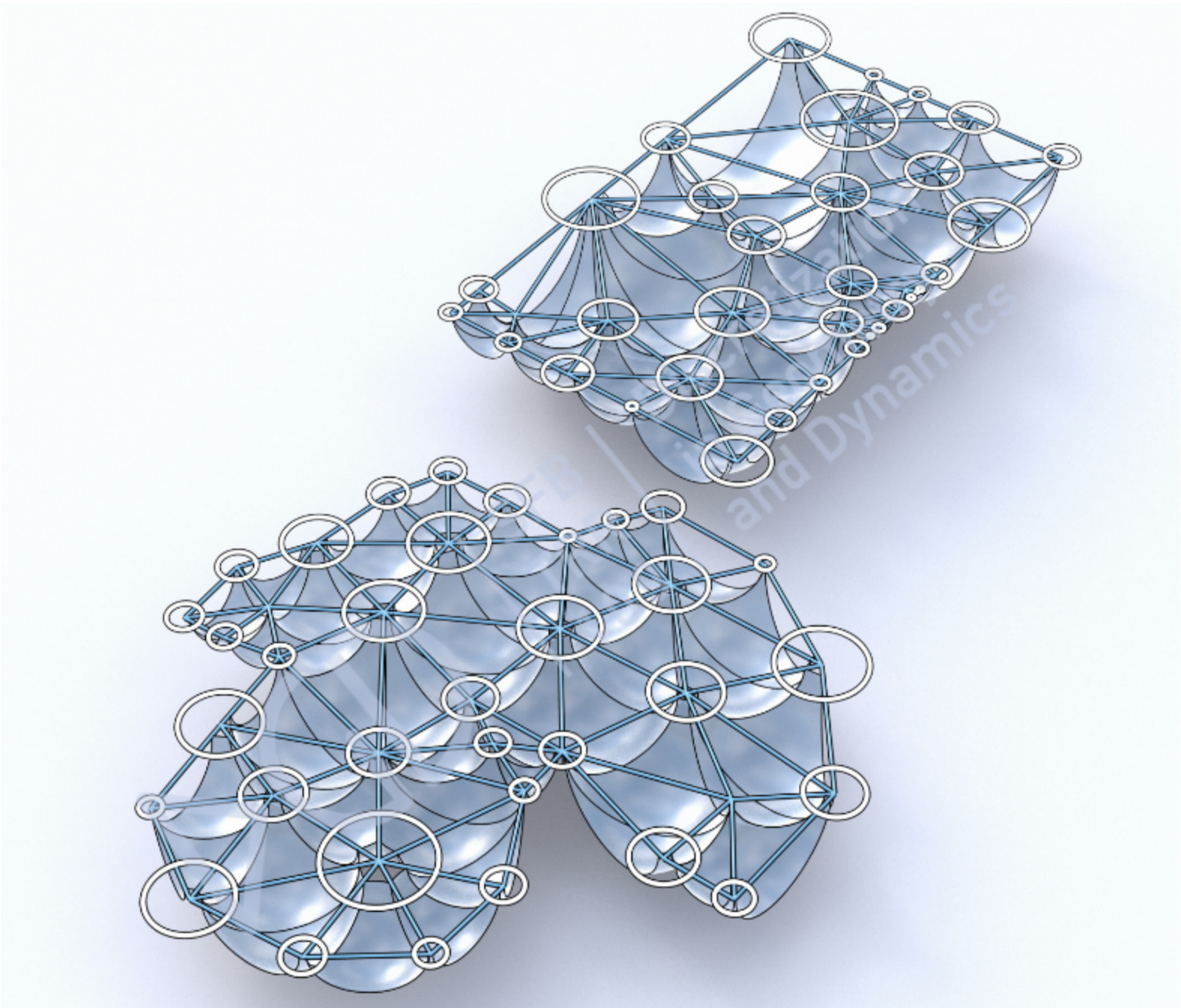

SEPTEMBER

Decorated Discrete Conformal Equivalence. Two decorated discrete domains are discrete conformal equivalent (DCE) if for each pair of triangles there is a Möbius transformation mapping the circles of one triangle to those of the other. A hyperbolic surface is associated to each discrete domain by interpreting the Euclidean plane as the ideal boundary of hyperbolic space. The decorated domains are DCE if and only if these surfaces are marked isometric.

Contributors. *Alexander I. Bobenko, Carl O. R. Lutz*

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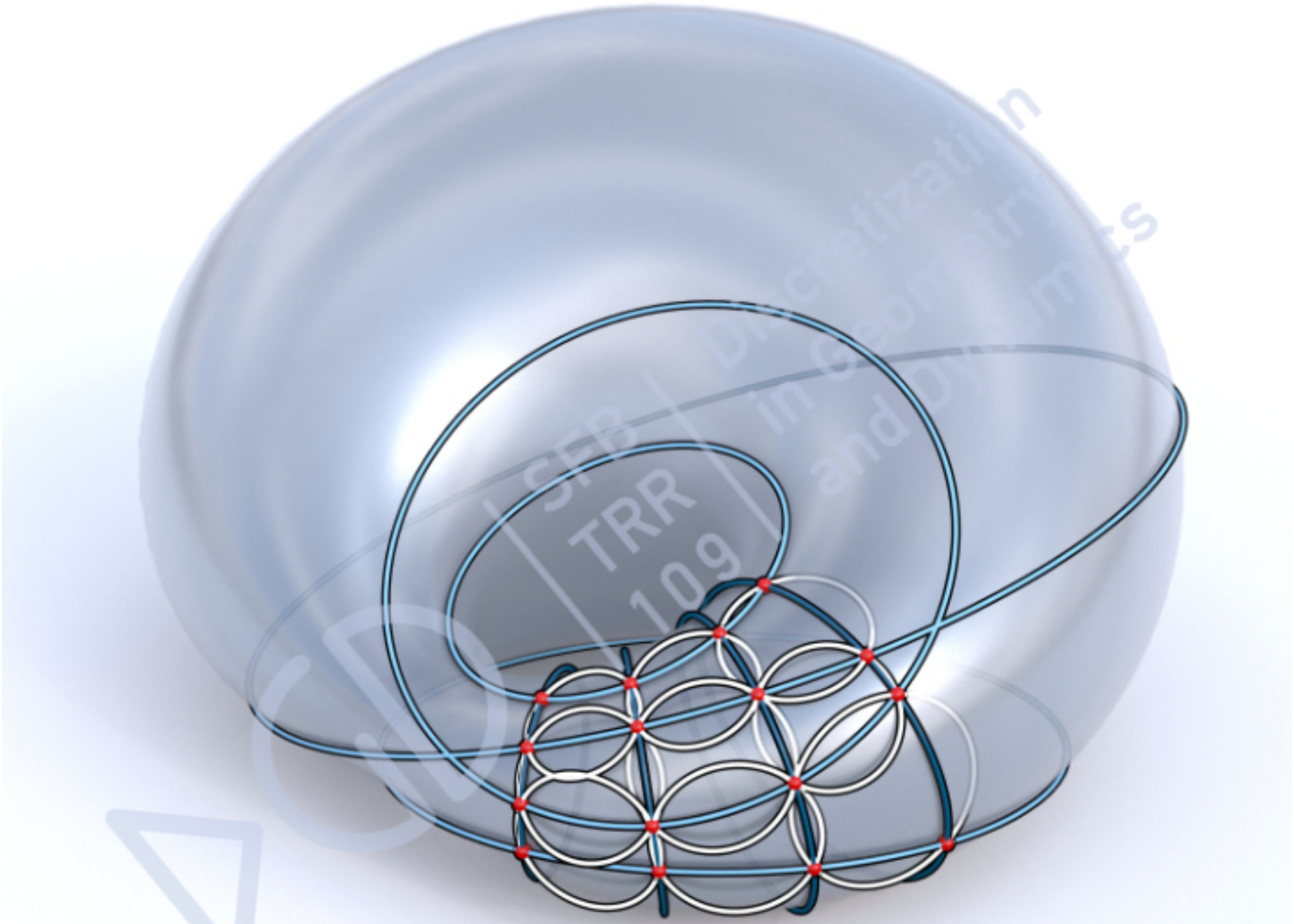

OCTOBER

Circular Nets with Circular Parameter Lines. The circles of the circular parameter lines are contained in a Darboux cyclide. By identifying Euclidean 3-space with a 3-sphere via stereographic projection, the sixteen vertices correspond to the sixteen vertices of a hypercube that is inscribed in the 3-sphere.

Contributors. *Alexander Y. Fairley, Théo Tyburn*

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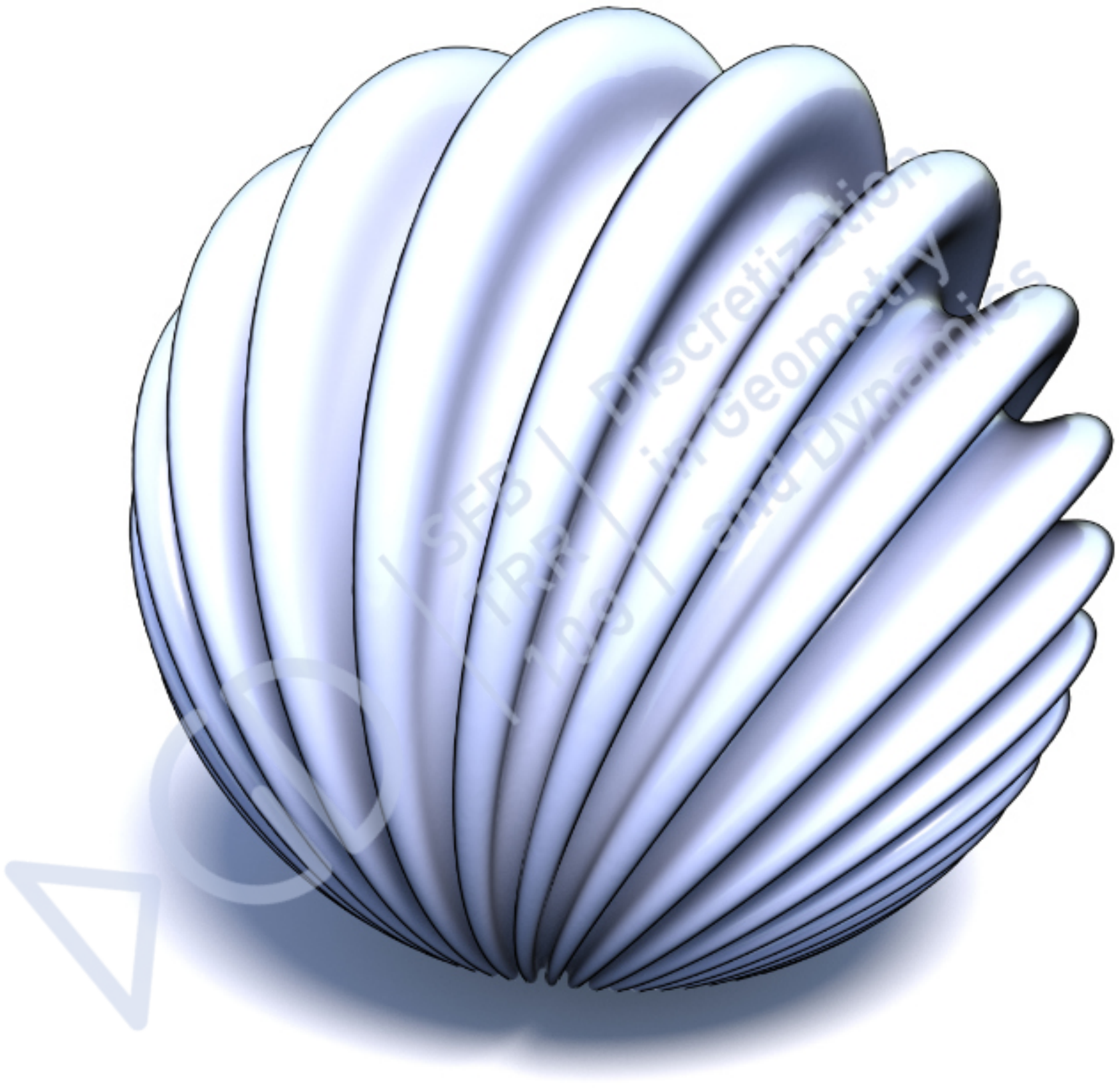

NOVEMBER

Differential Geometry — From Elastic Curves to Willmore Surfaces. This corrugated surface results from applying a Möbius transformation to a cylinder over a plane curve which is “free-elastic”. It not only is a so-called Willmore surface, but also elegantly combines many topics which can be covered in an introductory course on the differential geometry of curves and surfaces.

Contributors. *Ulrich Pinkall, Oliver Gross*

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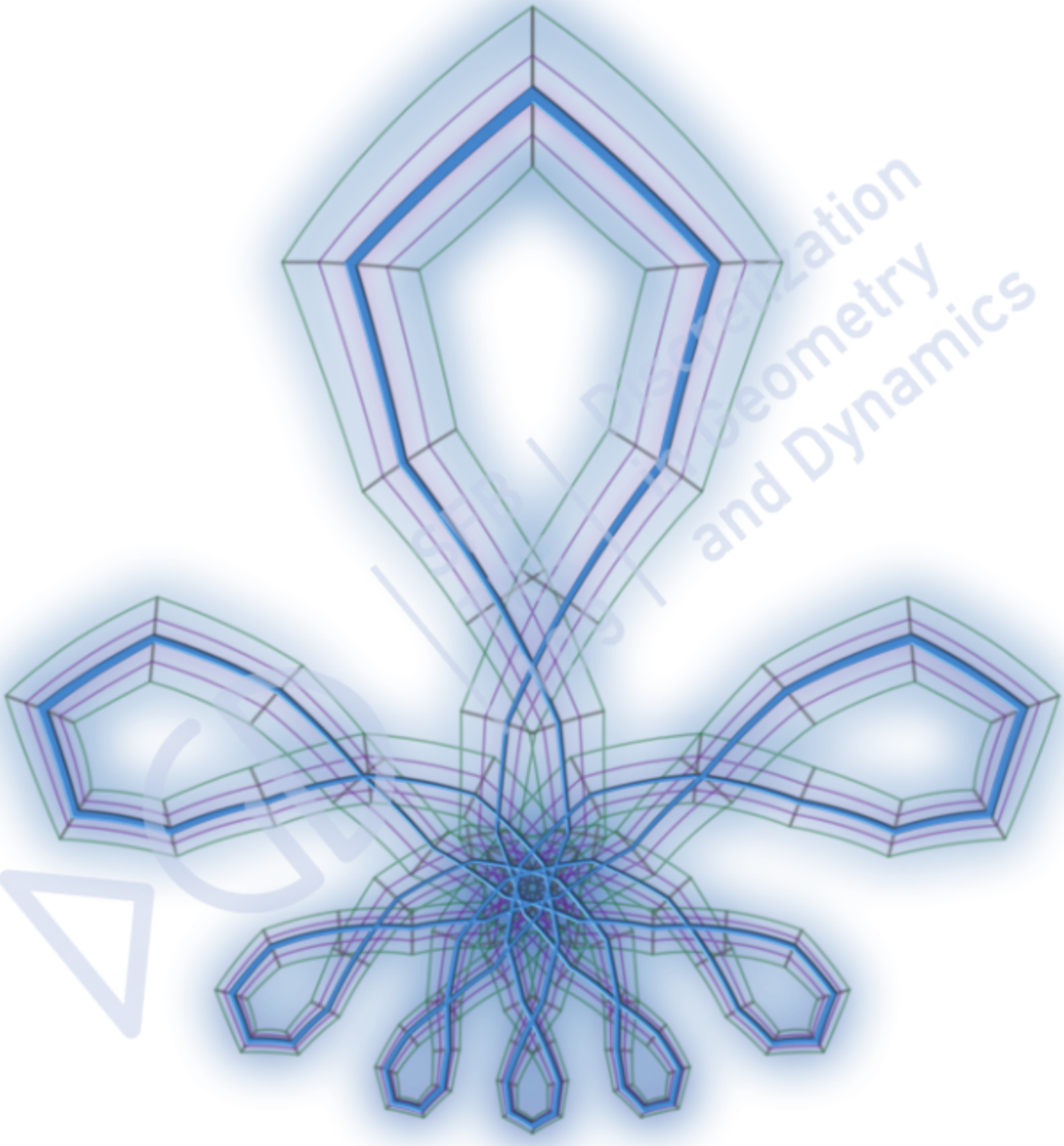

DECEMBER

Darboux Transformed Discrete Constrained Elastic Curves. Considered in the Poincaré disc model, the five curves have constant hyperbolic arc-length and satisfy a discrete version of the curvature equation characterizing smooth constrained elastic curves. The cross-ratio between two consecutive curves is constant.

Contributors. *Tim Hoffmann, Gudrun Szewieczek*

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JULY – DECEMBER

July

Motion from Shape Change. What do spermatozoa, snakes, stingrays, falling cats, astronauts and platform divers have in common? They all take specific sequences of poses in order to generate some kind of motion, that is translation and/or rotation. This project is concerned with computational aspects of geometric mechanics and how it allows for a unified algorithmic treatment of seemingly very different situations.

Reference.

Gross, O., Soliman, Y., Padilla, M., Knöppel, F., Pinkall, U., Schröder, P. (2023). Motion from Shape Change. *ACM Trans. Graph.*, 42(4), 107:1 – 107:11

August

Symmetric Tangling of Honeycomb Networks. Symmetric, elegantly entangled structures are curious objects that have found their way into the heart of the chemistry lab and the toolbox of constructive geometry. We considered the symmetric tangling of multiple 2-periodic honeycomb networks. We do this using a constructive methodology borrowing elements of graph theory, low-dimensional topology and geometry. The result is a wide-ranging enumeration of symmetric tangled honeycomb networks.

Reference.

Evans, E.M., Hyde, S.T. (2022) Symmetric Tangling of Honeycomb Networks. *Symmetry*, 14(9)

September

Decorated Discrete Conformal Equivalence. A decoration of a piecewise Euclidean surface is a choice of a circle about each of its vertices. We prove the corresponding discrete uniformization theorem, under the assumption that the circles do not intersect. The special case that all circles degenerate to points gives the known uniformization result for discrete conformal maps. We derive a variational principle and exhibit the close connection to canonical tessellations of hyperbolic surfaces and convex hyperbolic polyhedra.

Reference.

Bobenko, A.I., Lutz, C.O.R. (2023) Decorated discrete conformal maps and convex polyhedral cusps, *Preprint*, arXiv: 2305.10988 [math.GT]

October

Circular Nets with Circular Parameter Lines. Surfaces with two families of circular curvature lines are known as Dupin cyclides. Dupin cyclides are a subclass of Darboux cyclides. Circular nets are known to provide a discretization of curvature-line parametrizations. Remarkably, the circular net in the image of the month is contained in a Darboux cyclide that is not a Dupin cyclide. More generally, the research project develops a discretization of surfaces with spherical curvature lines.

Reference.

Bobenko, A.I., Fairley, A.Y. (2023) Circular nets, spherical parameter lines and terminating Laplace sequences, *Preprint*

November

Differential Geometry — From Elastic Curves to Willmore Surfaces. The study of “Differential Geometry of Curves and Surfaces” is of interest to mathematicians, physicists, and computer scientists alike. This project has resulted in a textbook suitable for beginners from a variety of fields. In addition to classical topics of differential geometry, this textbook pays special attention to topics that are relevant for practical applications, such as calculus of variations.

Reference.

Pinkall, U., Gross, O. (2023) *Differential Geometry — From Elastic Curves to Willmore Surfaces.* Compact Textbooks in Mathematics, Birkhäuser, Cham

December

Darboux transformed discrete constrained elastic curves. As recently shown, spherical curvature lines of smooth isothermic surfaces are Möbius images of constrained elastic curves in various space forms. Based on a novel discretization of constrained elastic curves (joint with Jannik Steinmeier), we investigate this phenomenon for discrete isothermic nets. Using Möbius inversions, we demonstrate how these nets can be reconstructed from sequences of Darboux transformed planar discrete constrained elastic curves.

Reference.

Hoffmann, T., Szewieczek, G. (in preparation) Discrete isothermic nets with spherical curvature lines.

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The central goal of the SFB/Transregio is to pursue research on the discretization of differential geometry and dynamics. In both fields of mathematics, the objects under investigation are usually governed by differential equations. Generally, the term "discretization" refers to any procedure that turns a differential equation into difference equations involving only finitely many variables, whose solutions approximate those of the differential equation.

The common idea of our research in geometry and dynamics is to find and investigate discrete models that exhibit properties and structures characteristic of the corresponding smooth geometric objects and dynamical processes. If we refine the discrete models by decreasing the mesh size they will of course converge in the limit to the conventional description via differential equations. But in addition, the important characteristic qualitative features should be captured even at the discrete level, independent of the continuous limit. The resulting discretizations constitutes a fundamental mathematical theory, which incorporates the classical analog in the continuous limit.

The SFB/Transregio 109 brings together scientists from the fields of geometry, dynamics and applications, to join forces in tackling the numerous problems raised by the challenge of discretizing their respective disciplines.

www.discretization.de

Impressum

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Discretization in Geometry and Dynamics

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