SFB TRR 109 Discretization in Geometry and Dynamics

Calendar 2022

CONTENTS & REFERENCES

JANUARY – JUNE

January

Periodic CMC Surfaces. New surfaces with constant mean curvature (CMC) are constructed by loop group factorization methods using the theory of integrable systems. This project focuses on periodic CMC surfaces, surfaces with Delaunay ends and on compact minimal surfaces in the three-dimensional sphere.

Reference.
Bobenko, A.I., Heller, S., Schmitt, N. (2021). Constant mean curvature surfaces based on fundamental quadrilaterals. *Math. Phys. Anal. Geom.*, 24(37)

February

Computational Design of Weingarten Surfaces. Weingarten surfaces are characterized by a functional relation between their principal curvatures. An optimization approach is used to find a Weingarten surface that is close to a given input design by aligning curvature isolines of the surface. The unknown functional relation then emerges as a result of the optimization. Several design studies illustrate how Weingarten surfaces define a versatile shape space for fabrication-aware exploration in freeform architecture.

Reference.
Pellis, D., Kilian, M., Pottmann, H., Pauly, M. (2021). Computational design of Weingarten surfaces. *ACM Trans. Graph.*, 40(4), 114:1 – 114:11

March

Compact Bonnet Pair. In this project, the researchers explicitly construct a pair of immersed tori in three dimensional Euclidean space that are related by a mean curvature preserving isometry. This resolves a longstanding problem on whether the metric and mean curvature function determine a unique compact surface. The construction is based on a relationship between Bonnet pairs and isothermic surfaces. In particular, the Bonnet pairs arise from isothermic surfaces with one family of planar curvature lines.

Reference.
Bobenko, A.I., Hoffmann, T., Sageman-Furnas, A.O. (2021). Compact Bonnet Pairs: isometric tori with the same curvatures. arXiv: 2110.06335 [math.DG]

April

Geometric Materials. Small-scale "kirigami" cut and fold patterns on a surface define rigid faces connected by hinges and turn the surface into a mechanical metamaterial. It is typically of negative Poisson ratio and morphs between different shapes. We use optimization to solve the fundamental computational challenge in this area, namely the targeted programming of a morph between given surfaces.

Reference.
Jiang, C., Rist, F., Wang, H., Wallner, J., Pottmann, H. (2021). Shape-morphing mechanical metamaterials. *Computer-Aided Design*, to appear

May

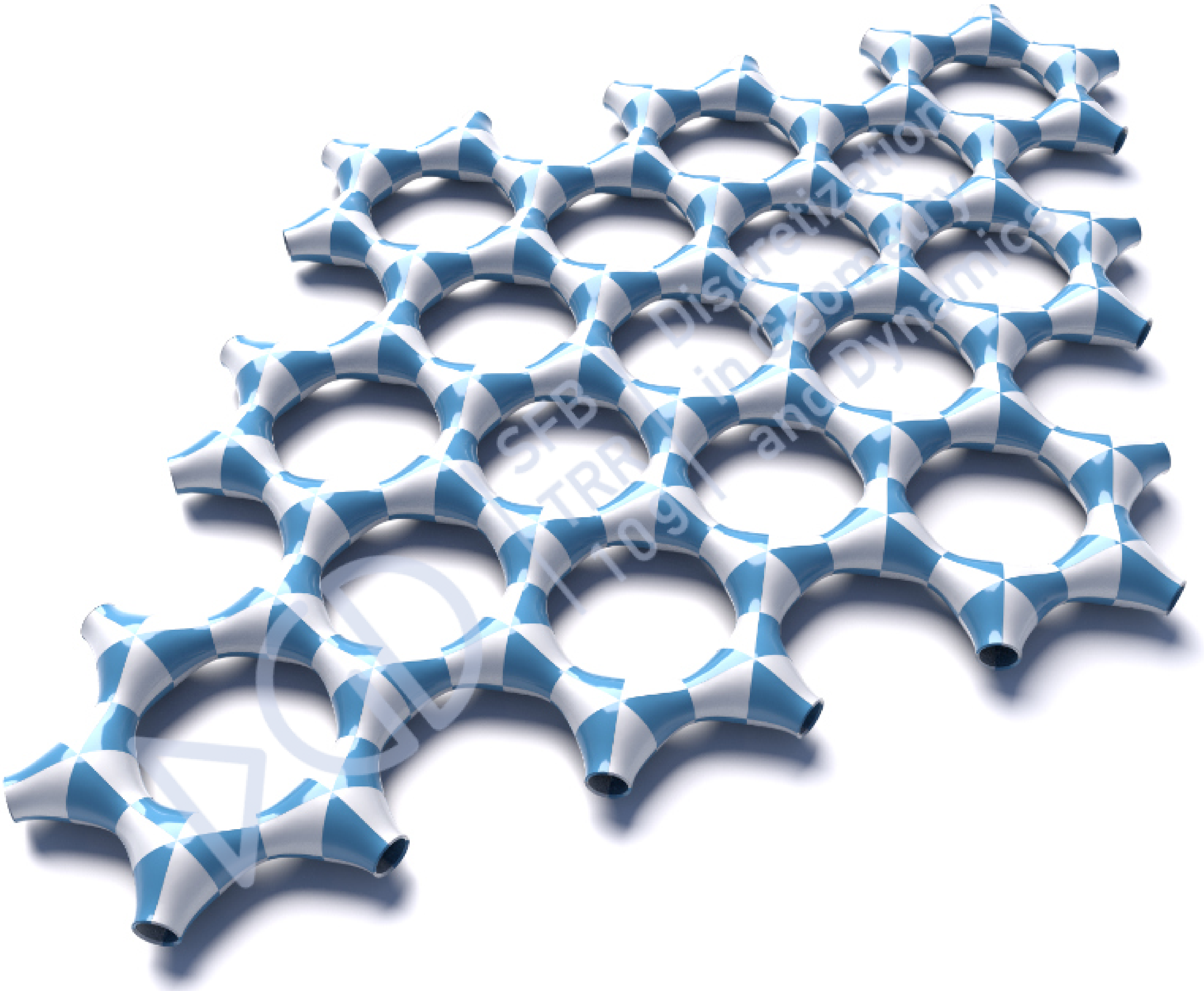
Conformally Twisted Tori. Although the surfaces in this picture of the month look smooth, in reality they are represented as a collection of triangles. Defining the conformal type (Riemann surface structure) of such a triangle mesh is important for applications. At the same time it is theoretically challenging. Numerically controlling the conformal type while optimizing quantities like global smoothness requires a precise understanding of the underlying discrete geometry.

Reference.
Soliman, Y., Chern, A., Diamanti, O., Knöppel, F., Pinkall, U., Schröder, P. (2021). Constrained Willmore Surfaces. *ACM Trans. Graph.*, 40(4), 112:1 – 112:17

June

Discrete CMC Surfaces. This research project focuses on construction of discrete surfaces with constant mean curvature built by touching discs. The surfaces are S-isothermic, they also possess touching spheres that intersect the discs orthogonally in their points of contact. Discrete CMC surfaces are recovered from their discrete Gauß map. The latter is given by an orthogonal ring pattern on a sphere.

Reference.
Bobenko, A. I., Hoffmann, T., Smeenk, N. (in preparation). Constant mean curvature surfaces from ring patterns: Geometry from combinatorics.



JANUARY

Periodic CMC Surfaces. This image shows a smooth doubly-periodic surface with constant mean curvature. It is constructed by loop group factorization methods using the theory of integrable systems. The surface is separated into congruent quadrilaterals by a pattern of curvature lines.

Contributors. *Alexander I. Bobenko, Sebastian Heller, Nicholas Schmitt*

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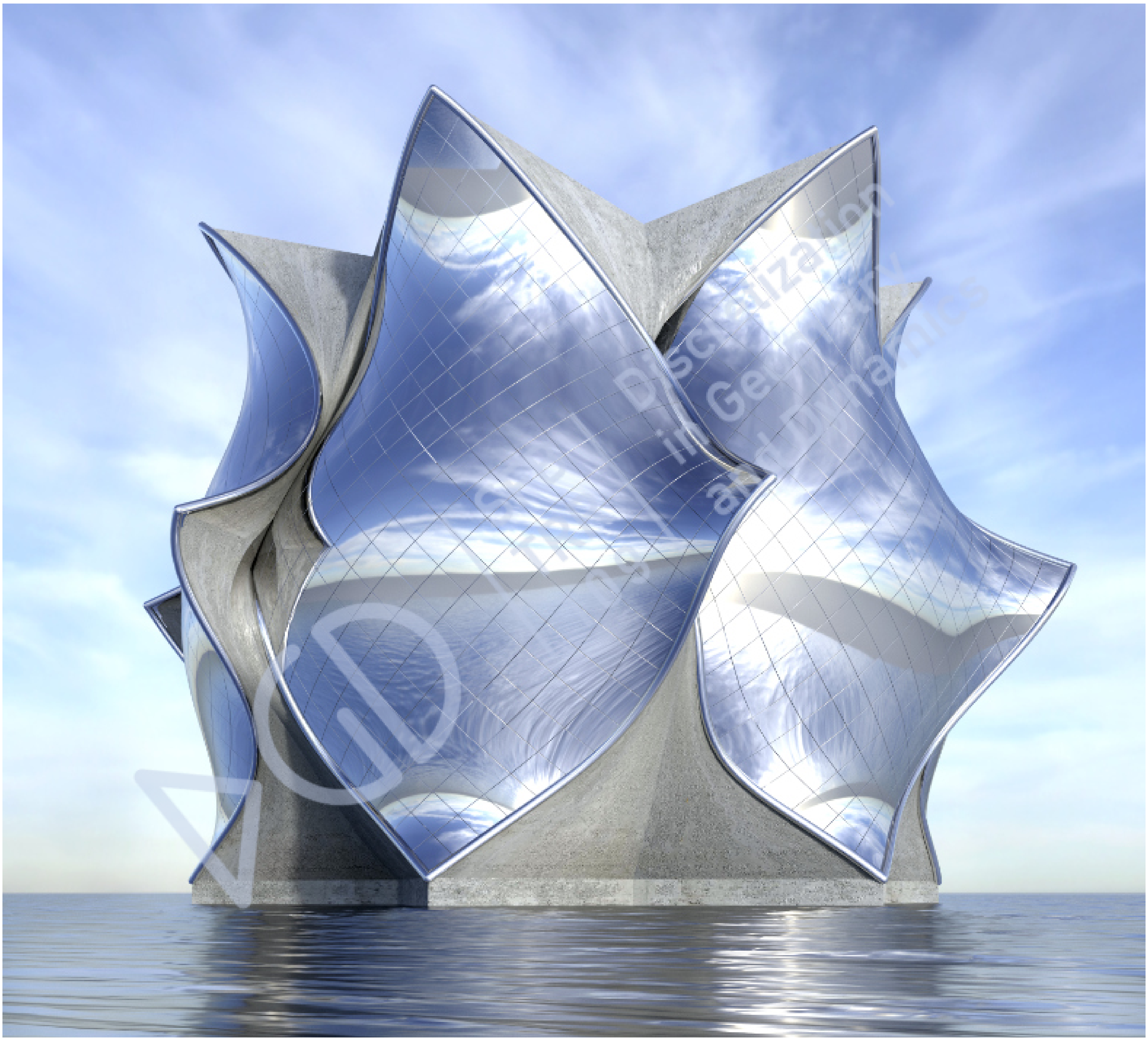

FEBRUARY

Computational Design of Weingarten Surfaces. Architectural design study resulting from an algorithm that approximates a free-form surface by a Weingarten surface. Having only a one parameter family of different curvature elements, Weingarten surfaces enable cost effective paneling solutions through mould re-use.

Contributors. *Davide Pellis, Martin Kilian, Helmut Pottmann, Mark Pauly*

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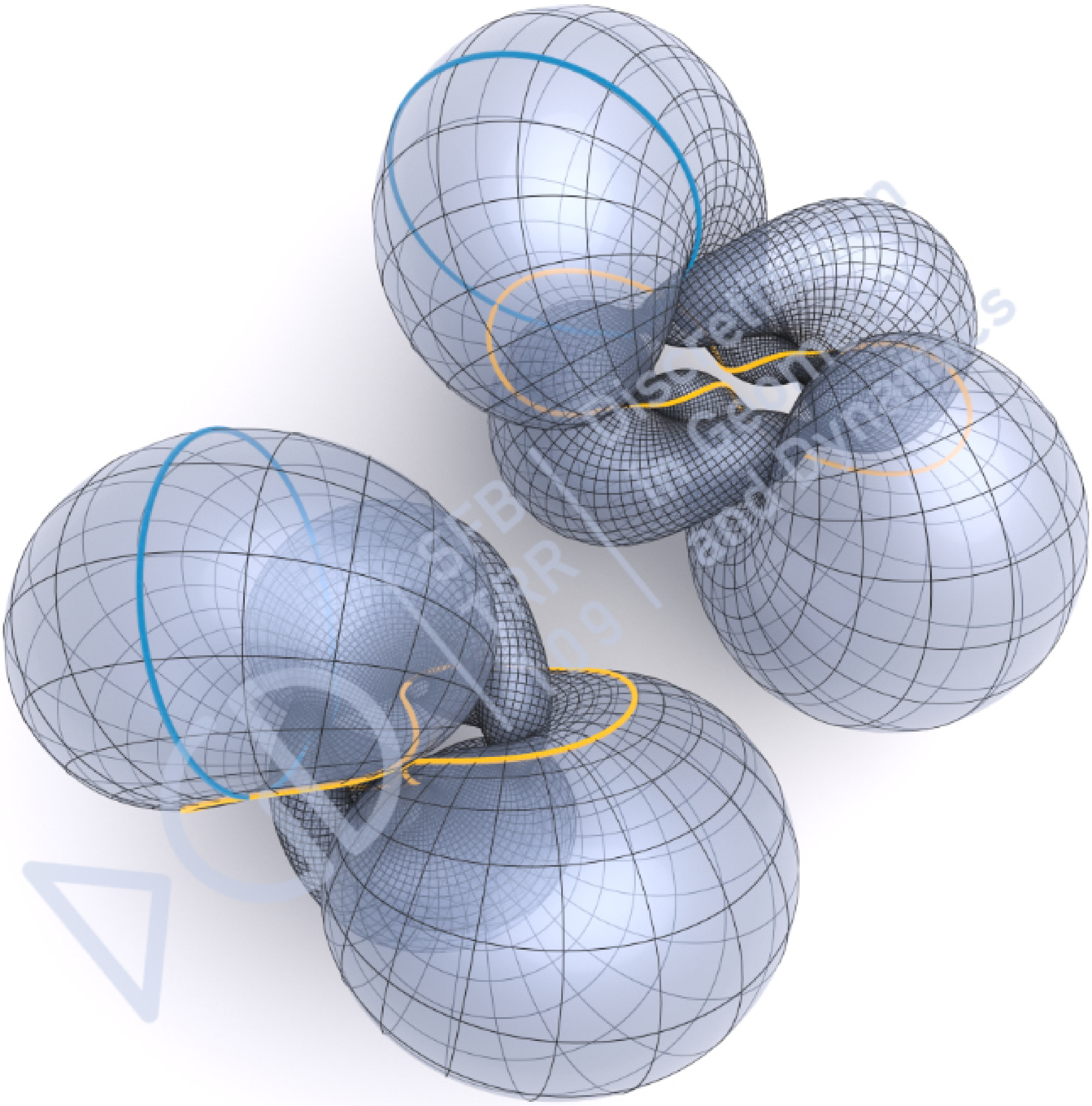

MARCH

Compact Bonnet Pair. Two non-congruent immersed tori that are related by a mean curvature preserving isometry. The corresponding orange lines are non-congruent. These are the first examples of compact Bonnet pairs.

Contributors. *Alexander I. Bobenko, Tim Hoffmann, Andrew O. Sageman-Furnas*

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
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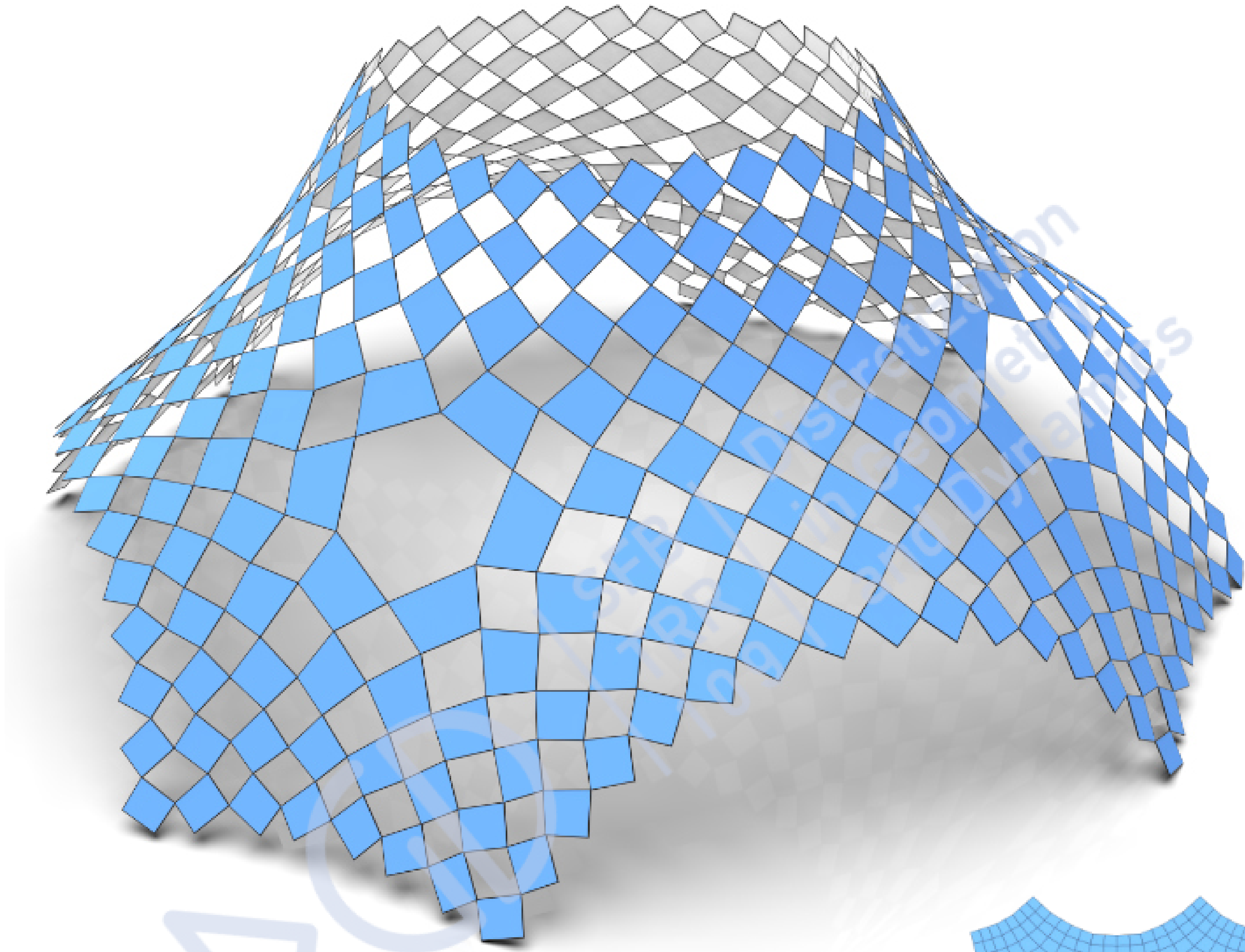

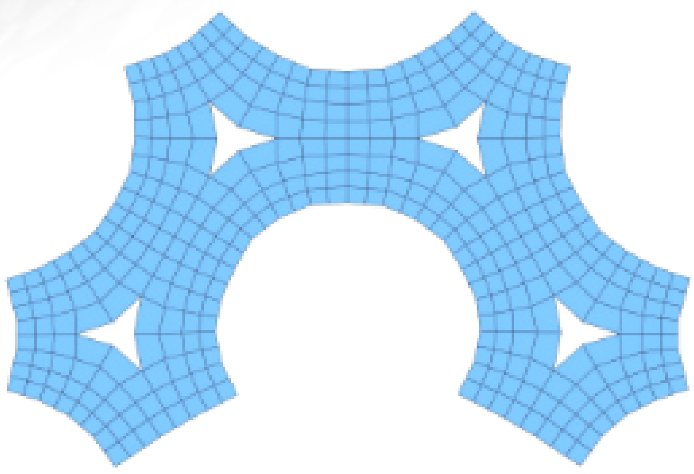

APRIL

Geometric Materials. Incisions in flat paper (lower right) along the edges of a quad mesh define a geometric metamaterial deploying to a discrete surface in space (upper left). Here the near-square shape of faces and the fact of full expansion together imply that the morph from the open to the closed state approximates a constant mean stretch deformation.

Contributors. *Caigui Jiang, Florian Rist, Hui Wang, Johannes Wallner, Helmut Pottmann*

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MAY

Conformally Twisted Tori. A sequence of tori which are conformally twisted. Each one is an optimal model for its underlying Riemann surface in the sense that it is optimally smooth. This means that these surfaces are critical points (although not minimizers) of the Willmore functional among conformally deformed versions of them.

Contributors. *Yousuf Soliman, Albert Chern, Olga Diamanti, Felix Knöppel, Ulrich Pinkall, Peter Schröder*

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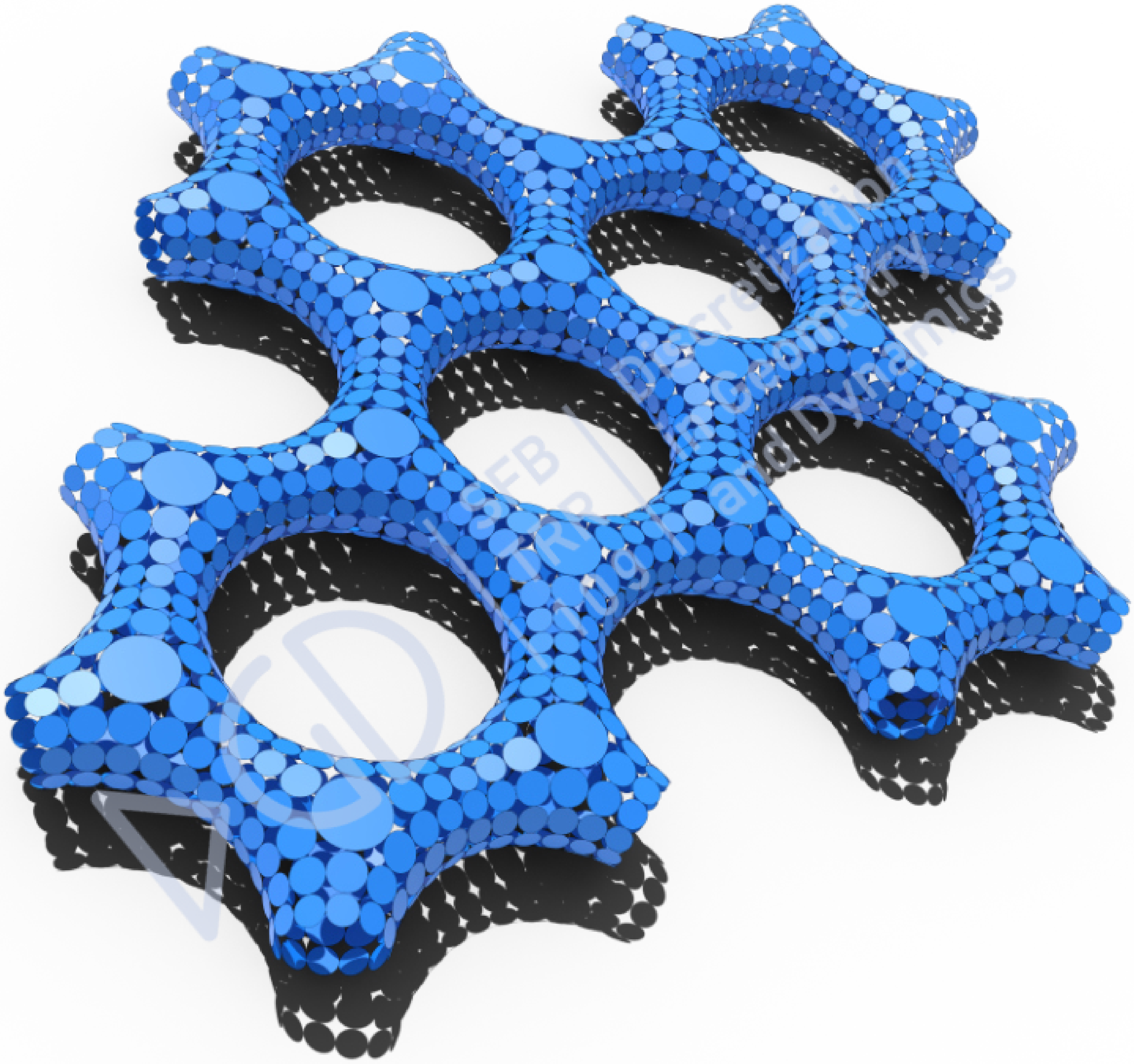

JUNE

Discrete CMC Surfaces. This image shows a discrete analog of the smooth doubly-periodic surface with constant mean curvature shown in January. It is comprised of touching discs which can be computed from the corresponding orthogonal ring patterns on the sphere.

Contributors. *Alexander I. Bobenko, Tim Hoffmann, Thilo Rörig, Nina Smeenk*

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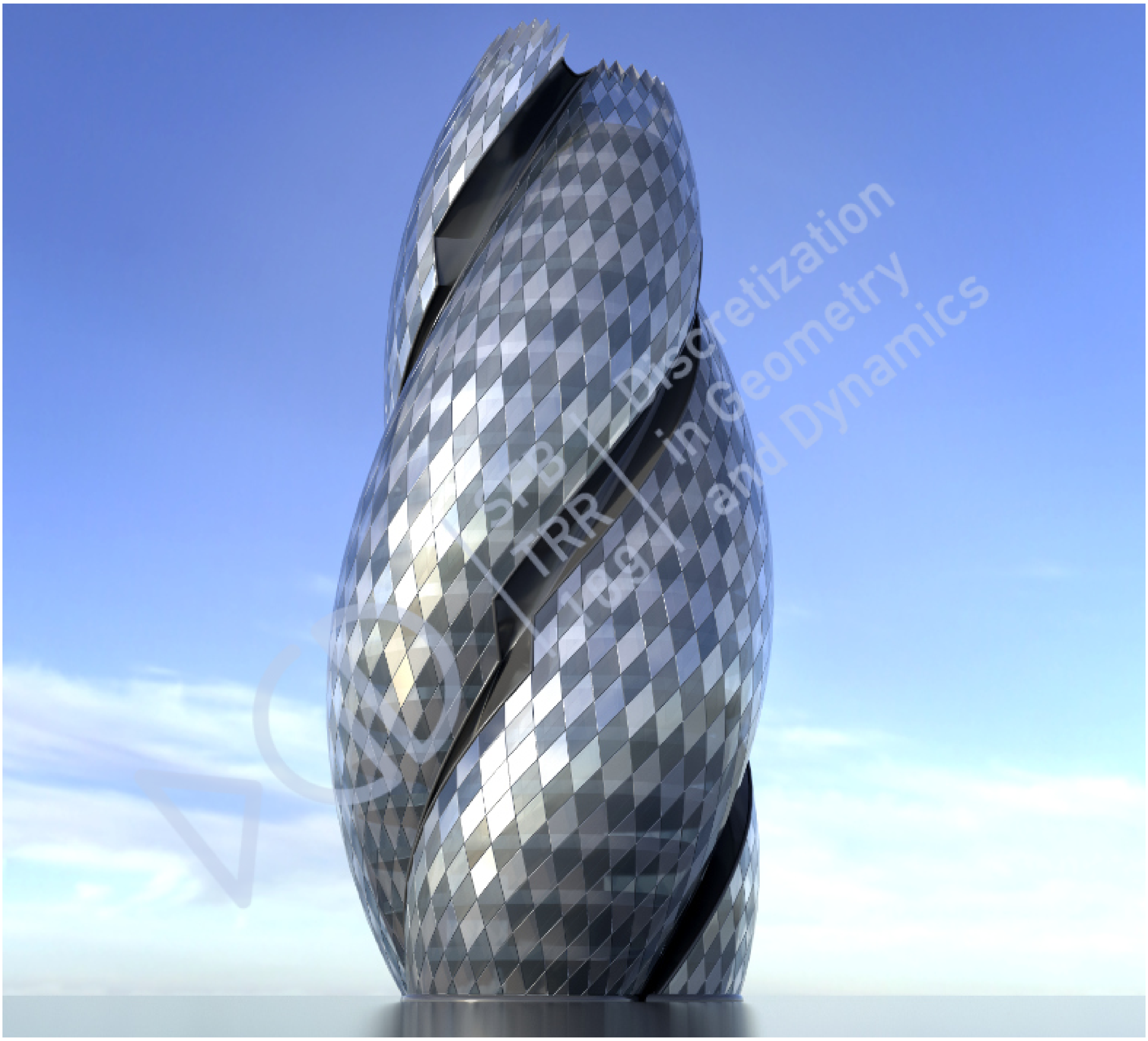

JULY

Principal Symmetric Meshes. An architectural rendering of a tower consisting of three facade surfaces, each of them with a panelization along the discrete characteristic conjugate net. All faces were modeled as planar dimmed glass panels.

Contributors. *Davide Pellis, Hui Wang, Martin Kilian, Florian Rist, Helmut Pottmann, Christian Müller*

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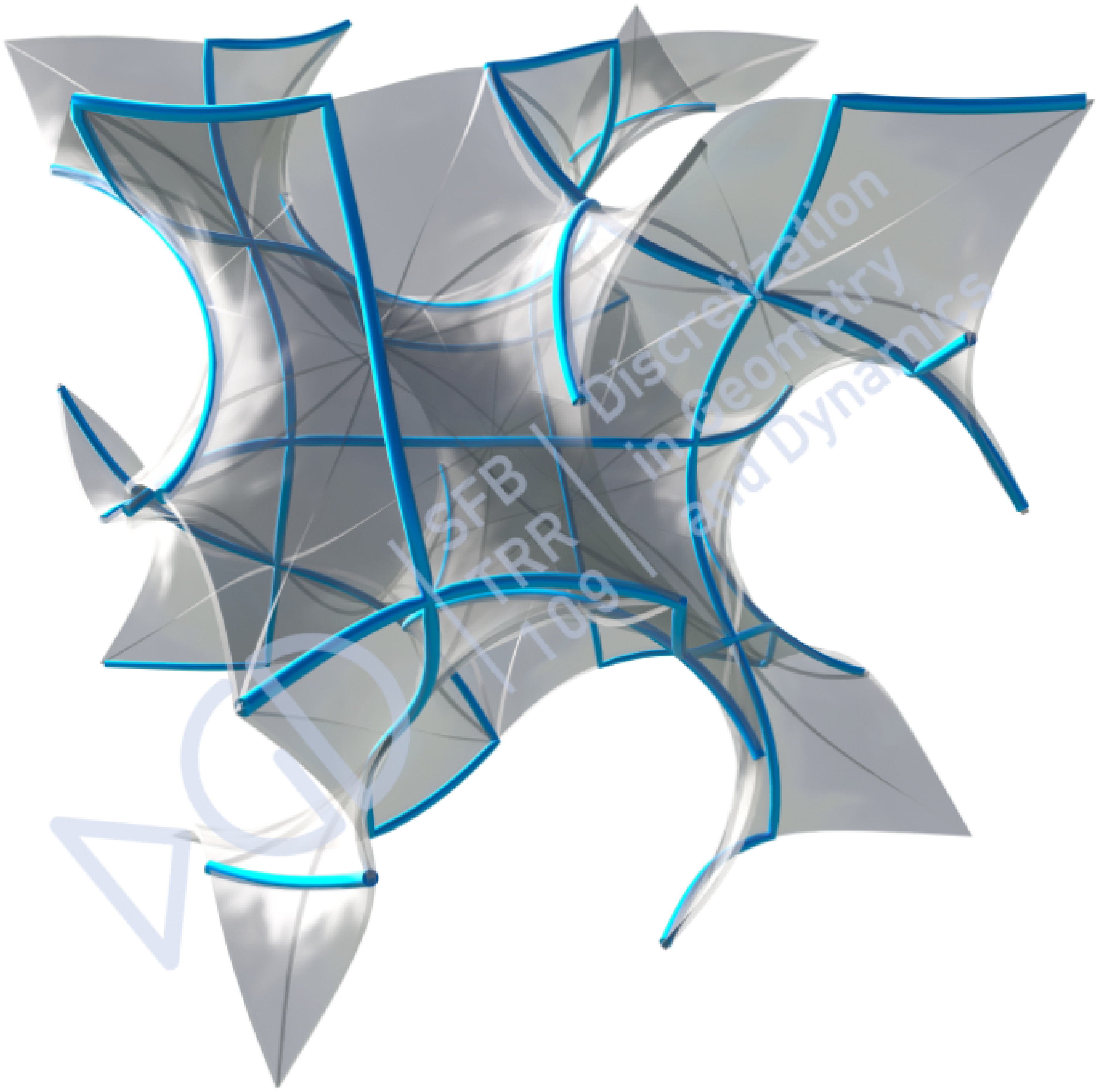
DGD SFB TRR 109 Discretization in Geometry and Dynamics



AUGUST

Edge Graph of a Tiling on the Gyroid. A highly symmetric 3-periodic entangled net on the surface of the gyroid triply-periodic minimal surface. The gyroid is ubiquitous, appearing both naturally inside cells, butterfly wing scales, and bird feathers, as well as as a target of synthesis in materials science. It fills out space and embodies hyperbolic geometry in our 3-dimensional world.

Contributors. *Benedikt Kolbe, Myfanwy E. Evans*

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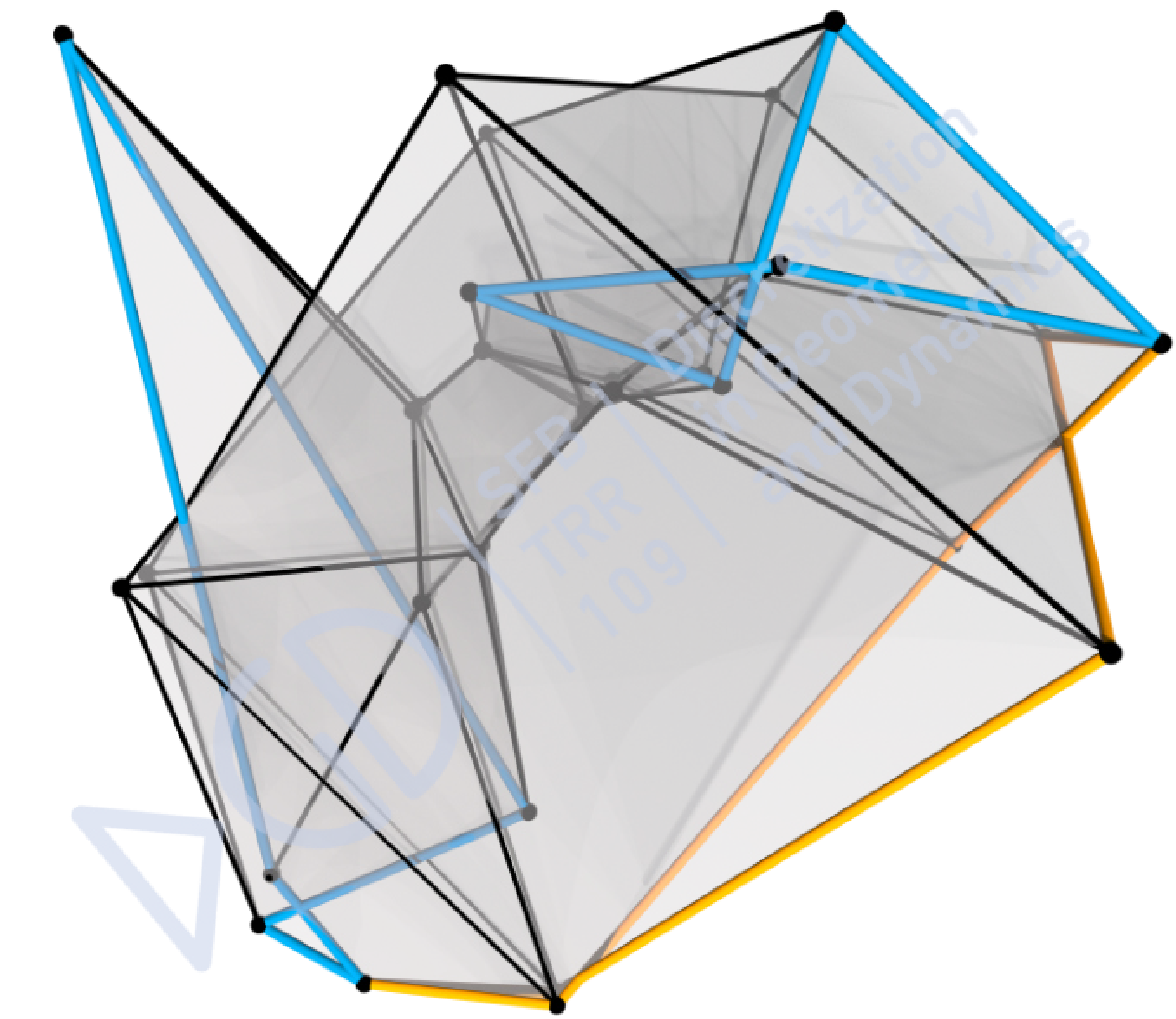

SEPTEMBER

Discrete Isothermic Torus. The first numerical example of a discrete isothermic torus that gives rise to a discrete compact Bonnet pair of tori. This extremely coarse numerical example on a 5×7 lattice exhibits the remarkable property that led to the discovery of smooth compact Bonnet pairs: it has one family of planar curvature lines (blue).

Contributors. *Alexander I. Bobenko, Tim Hoffmann, Andrew O. Sageman-Furnas*

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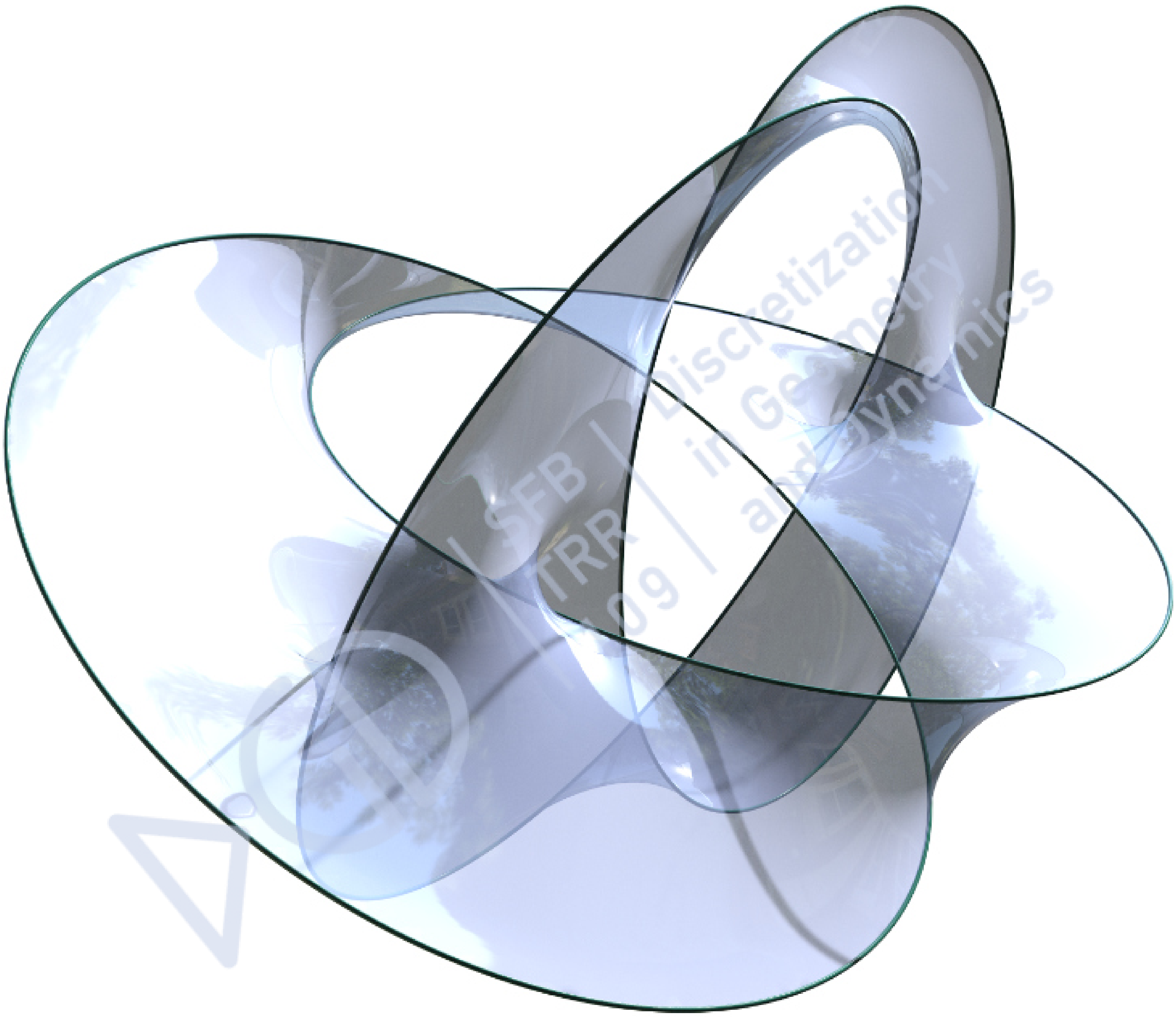

OCTOBER

Computing Minimal Surfaces with Differential Forms. A minimal surface bordered by five randomly positioned rings. The computation uses geometric measure theory to convexify the classical Plateau minimal surface problem and searches for minimal surfaces across all topologies.

Contributors. *Stephanie Wang, Albert Chern*

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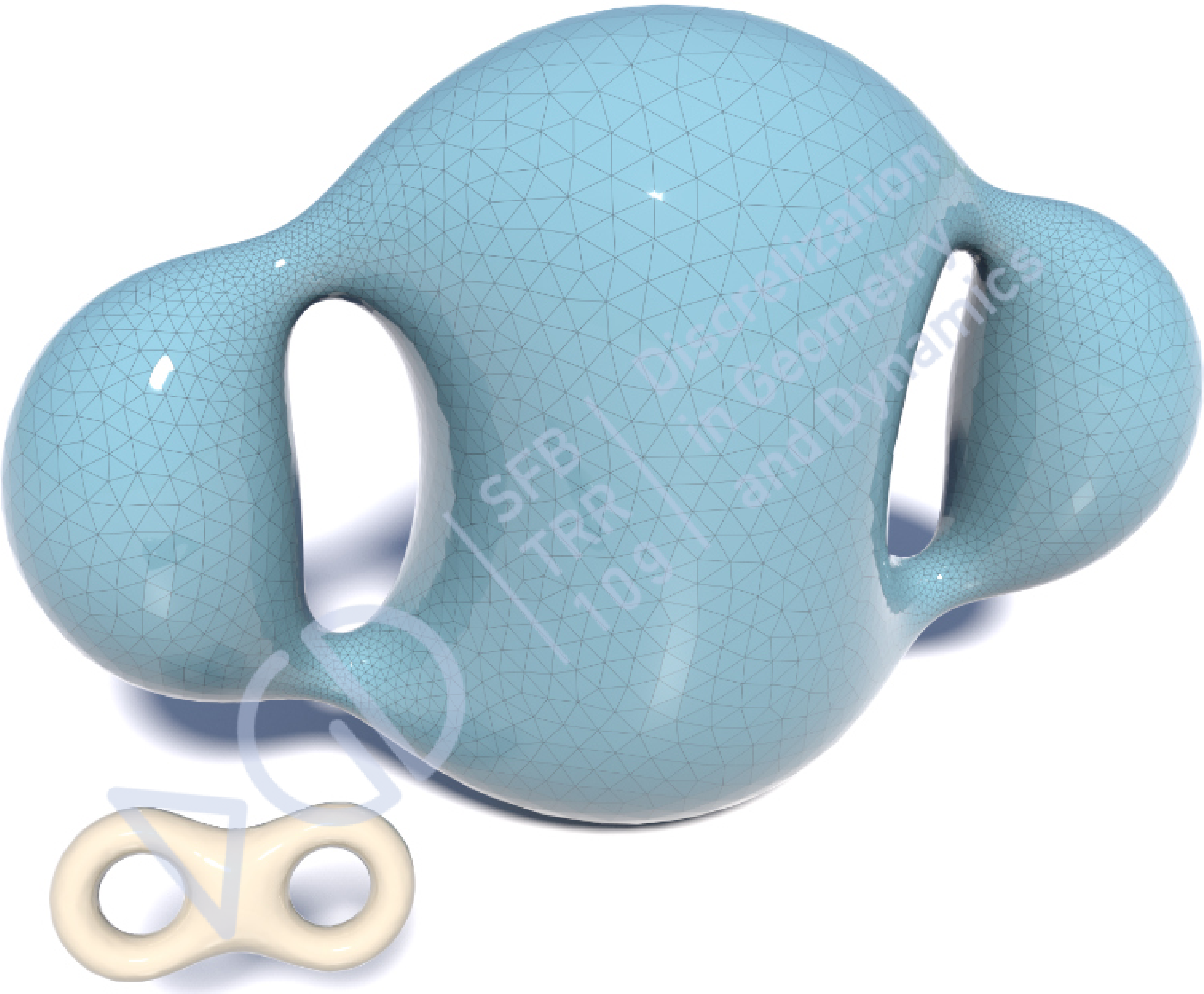

NOVEMBER

Discrete Constrained Willmore Surfaces. The white surface shows the initial surface of a geometric optimization problem: a sphere with two handles. The blue surface shows a deformed version that arises as the result of minimizing the integral of the squared mean curvature (Willmore functional). The minimization happens subject to the constraint that the intrinsic geometry of the surface suffers no angle distortion.

Contributors. *Yousuf Soliman, Albert Chern, Olga Diamanti, Felix Knöppel, Ulrich Pinkall, Peter Schröder*

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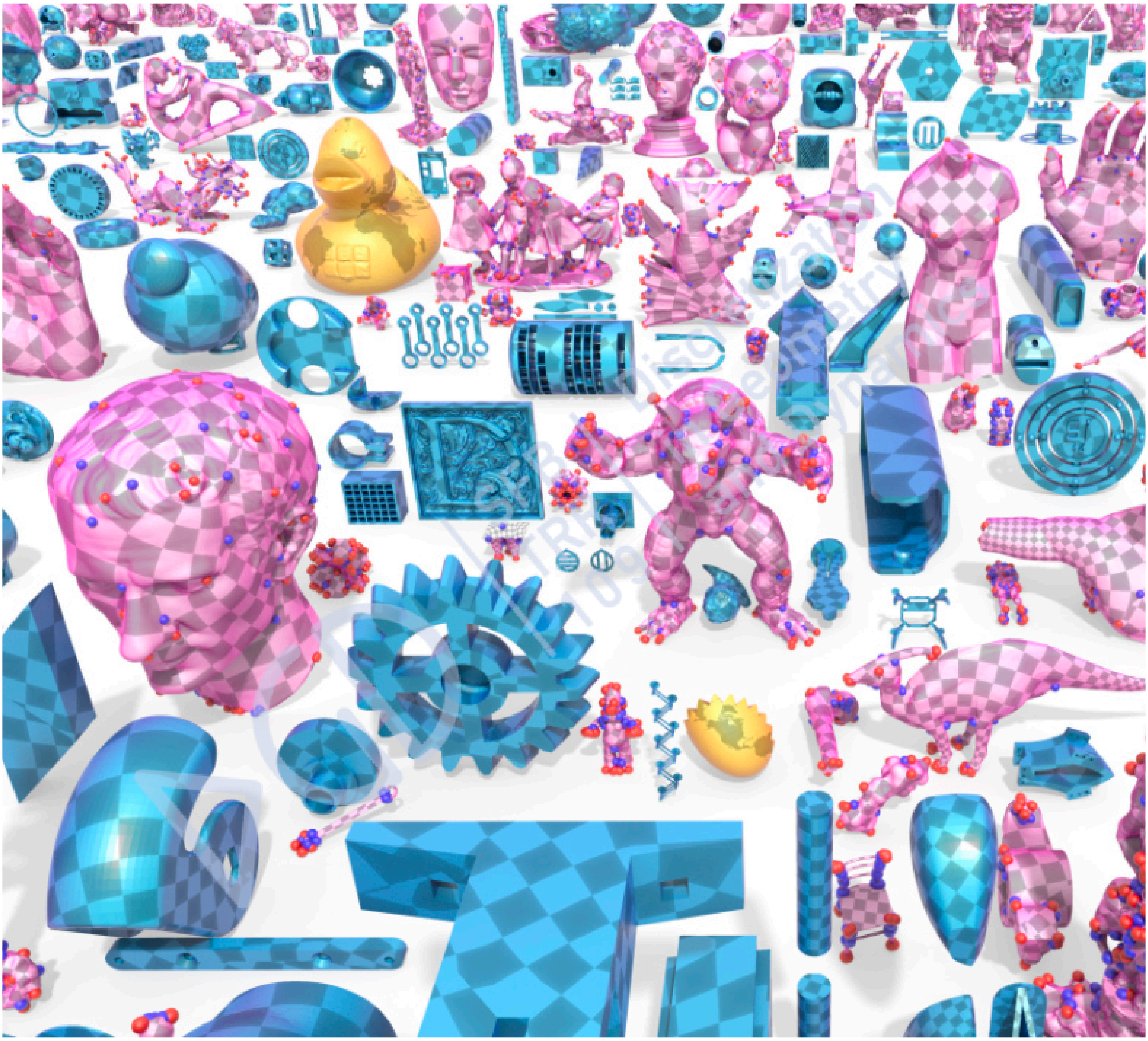

DECEMBER

Discrete Conformal Equivalence. A collage of texture maps that are conformal in a discrete sense. The algorithm used to compute them works even for near-degenerate triangulations (turquoise), extremely difficult configurations of cone singularities (magenta), and it can produce maps to the sphere (yellow).

Contributors. *Mark Gillespie, Boris Springborn, Keenan Crane*

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CONTENTS & REFERENCES

JULY – DECEMBER

July

Principal Symmetric Meshes. The isolines of principal symmetric surface parametrizations run symmetrically to the principal directions. Such nets are useful for various applications in the context of fabrication and architectural design. Their discretizations come naturally with a family of spheres, the so-called Meusnier spheres. Controlling their radii and the intersection angles of isolines facilitates tasks such as generating Weingarten surfaces including constant mean curvature surfaces and minimal surfaces.

Reference.

Pellis, D., Wang, H., Killian, M., Rist, F., Pottmann, H., Müller, C. (2020). Principal Symmetric Meshes. *ACM Trans. Graph.*, 39(4), 127:1 – 127:17

August

Edge Graph of a Tiling on the Gyroid. Motivated by its mechanical and optical properties and its ubiquity, the gyroid triply-periodic minimal surface serves as an important example for a construction technique that enumerates periodic arrays of networks as graphs embedded on hyperbolic surfaces with prescribed symmetries. The current research has resulted in the development of isotopic tiling theory for hyperbolic surfaces, which casts a theoretical framework that allows for a systematic treatment of related problems.

Reference.

Kolbe, B., Evans, M.E. (2020). Isotopic tiling theory of hyperbolic surfaces. *Geometriae Dedicata*, 212, 177 – 205

September

Discrete Isothermic Torus. Discrete Differential Geometry studies structure preserving discretizations of the smooth theory. This is exemplified by the computational discovery of discrete compact Bonnet pairs. Since the discretization is structure preserving it was possible to recover important properties of the corresponding isothermic tori, which initiated the work on constructing smooth compact Bonnet pairs.

Reference.

Bobenko, A.I., Hoffmann, T., Sageman-Furnas, A.O. (2021), Compact Bonnet Pairs: isometric tori with the same curvatures. arXiv: 2110.06335 [math.DG]

October

Computing Minimal Surfaces with Differential Forms. We describe a new algorithm for solving Plateau's classical minimal surface problem: find a surface of minimal area bordering an arbitrarily prescribed curve. Using the language of differential forms we recast the boundary constraint into a linear differential equation, and the surface area functional becomes the mass norm. This representation convexifies the optimization problems and allows us to compute minimal surfaces across all possible topologies.

Reference.

Wang, S., Chern, A. (2021). Computing Minimal Surfaces with Differential Forms. *ACM Trans. Graph.*, 40(4), 113:1 – 113:14

November

Discrete Constrained Willmore Surfaces. It is known that every Riemann surface can be realized without angle distortion as a surface in space. On the other hand, finding an optimal realization for each Riemann surface (with only the necessary amount of curvature, without superfluous bumps) poses a challenging theoretical problem. Numerical algorithms (based on discrete differential geometry) and experiments are an important tool for gaining insights into the geometry of such optimal realizations.

Reference.

Soliman, Y., Chern, A., Diamanti, O., Knöppel, F., Pinkall, U., Schröder, P. (2021). Constrained Willmore Surfaces. *ACM Trans. Graph.*, 40(4), 112:1 – 112:17

December

Discrete Conformal Equivalence. A discrete conformal map acts on a triangle mesh by multiplying all edge lengths with scale factors associated to the vertices. This may violate triangle inequalities, but the triangulation is "repaired" using a Delaunay edge flip algorithm with non-standard Ptolemy flips. The theory of discrete conformal maps is closely connected with hyperbolic geometry. Convex variational principles provide efficient methods for computing discrete conformal maps.

Reference.

Gillespie, M., Springborn, B., Crane, K. (2021). Discrete conformal equivalence of polyhedral surfaces. *ACM Trans. Graph.*, 40(4), 103:1 – 103:20

The central goal of the SFB/Transregio is to pursue research on the discretization of differential geometry and dynamics. In both fields of mathematics, the objects under investigation are usually governed by differential equations. Generally, the term "discretization" refers to any procedure that turns a differential equation into difference equations involving only finitely many variables, whose solutions approximate those of the differential equation.

The common idea of our research in geometry and dynamics is to find and investigate discrete models that exhibit properties and structures characteristic of the corresponding smooth geometric objects and dynamical processes. If we refine the discrete models by decreasing the mesh size, they will of course converge in the limit to the conventional description via differential equations. But in addition, the important characteristic qualitative features should be captured even at the discrete level, independent of the continuous limit. The resulting discretizations constitute a fundamental mathematical theory, which incorporates the classical analog in the continuous limit.

The SFB/Transregio 109 brings together scientists from the fields of geometry, dynamics and applications to join forces in tackling the numerous problems raised by the challenge of discretizing their respective disciplines.

www.discretization.de

Impressum

Collaborative Research Centre/Transregio 109
Discretization in Geometry and Dynamics

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