Planar quad layout on NURBS-surfaces from symmetric conjugate curves

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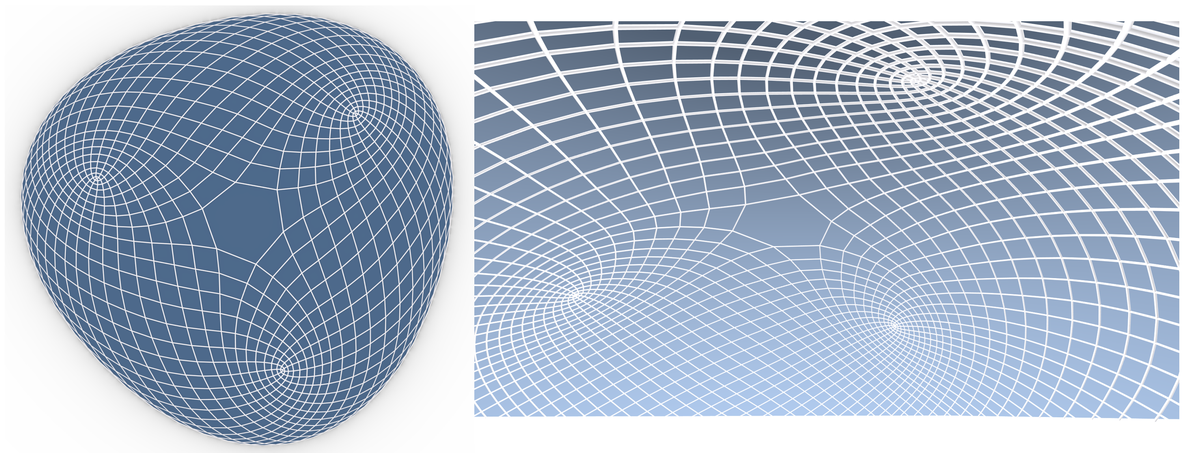
From conjugate directions to planar quads

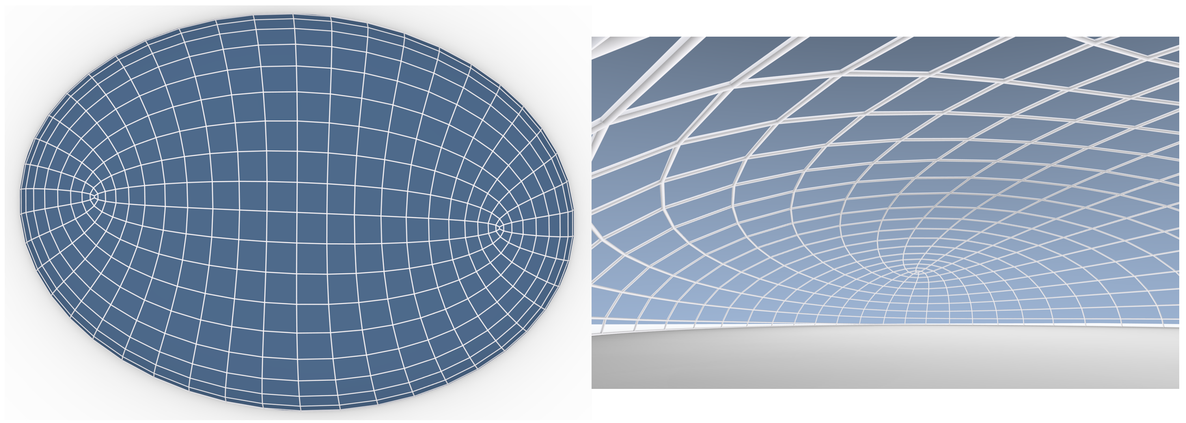
Our goal is to create meshes on surfaces consisting of planar quadrilaterals. On a surface, a network of curves along conjugate directions (see **(a)** and **(b)**) leads to a mesh with almost planar quads. For NURBS-surfaces we developed software to generate suitable networks using symmetries with respect to curvature directions (see **(c)** and **(d)**). We present surface meshes that consist of planar quads only and exhibit particularly nice geometries.

Triangular Pavillion



A mesh created from a NURBS-surface based on triangular control points. The curves follow conjugate directions that are symmetric with respect to the curvature direction **(c)**.

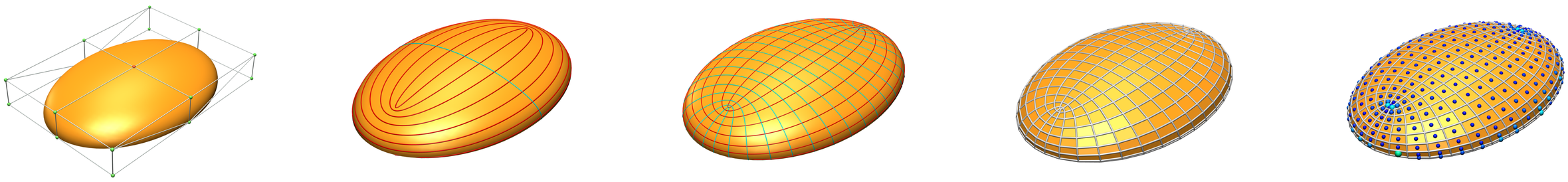
Ellipsoid Pavillion



In contrast to the Triangular Pavillion we changed the symmetry by rotating the curvature direction about $41°$ **(d)**. A key feature is that a **single** curve (following the first direction) is sufficient to generate a network of conjugate lines.

Workflow

(1) **(2)** **(3)** **(4)** **(5)**



(1) We start with an arbitrary NURBS-surface and select a point. **(2)** We choose angle α and obtain a curve on the surface. **(3)** Generating many curves leads to a network of conjugate lines. **(4)** The resulting mesh consists of almost planar quads. **(5)** If we are not satisfied with the planarity we optimize to obtain a planar quad mesh.

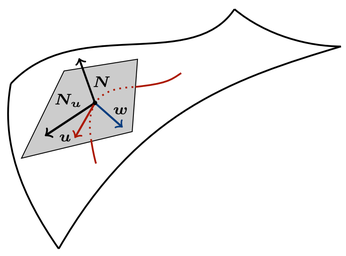
Conjugate curves

On a surface the direction of maximal curature v_{max} (black) is the direction in which the surface is curved the most. At selected points we choose an angle to rotate v_{max} and determine the appropriate symmetric conjugate directions. Our software creates the corresponding curves (red and blue).

Conjugate directions

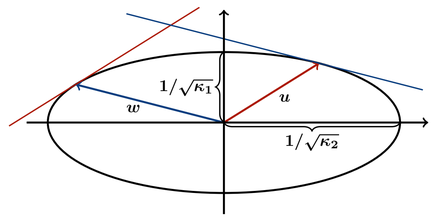
A direction w is *conjugate* to a given direction u if the directional derivative N_u of the normal vector N in direction u and w form a right angle.

(a)



Dupins Indicatrix

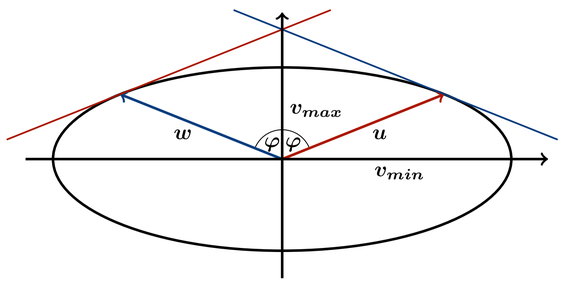
(b)



In case of positive Gauß curvature the *Dupin indicatrix* is an ellipse. A direction w is *conjugate* to u, if the line at u, that is tangent to the ellipse, is parallel to w, and vice-versa.

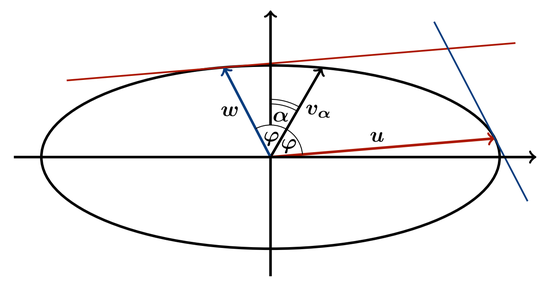
Symmetric conjugate directions

(c)



We consider conjugate directions u and w that are symmetric with respect to the curvature direction v_{max} (left), or, more generally, symmetric with respect to a direction v_α defined by the angle α to the curvature direction (right).

(d)



VaryLab - Discrete Surface Optimization

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All examples created with VaryLab http://www.varylab.com